CISCE VIRTUAL LEARNING SERIES

LESSON: MATHEMATICS

TRIGONOMETRIC IDENTITIES (SESSION 2)

October 19th, 2020

Response to Questions posed by students during the live Lesson:

S.No.	Questions	Answers
1.	Prove that $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$. This sum is proved by taking LHS. How will we prove the sum from the RHS?	Yes you may. Prove that $\sqrt{\frac{1-cos\,A}{1+cos\,A}} = \frac{sinA}{1+cosA}$ RHS = $\frac{sinA}{1+cosA}$ = $\sqrt{\frac{sin^2A}{(1+cosA)^2}}$ = $\sqrt{\frac{1-cos^2A}{(1+cosA)^2}}$ = $\sqrt{\frac{(1+cosA)(1-cosA)}{(1+cosA)(1+cosA)}}$ = $\sqrt{\frac{1-cos\,A}{1+cos\,A}}$ *LHS*
2.	*Prove that* $sec^4\theta - sec^2\theta = tan^4\theta + tan^2\theta.$ *Can we prove this identity by transferring a term from LHS to RHS or RHS to LHS?*	When proving an identity, it is advisable to start from one side and prove equal to the other side. Follow the rule for all sums. *Prove that* $sec^4\theta - sec^2\theta = tan^4\theta + tan^2\theta$ LHS= $sec^2\theta(sec^2\theta - 1)$ = $(1 + tan^2\theta)tan^2\theta$ = $tan^2\theta + tan^4\theta$ = $tan^4\theta + tan^2\theta$ = RHS Proved

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3.	How do we solve a problem like $sin^2A - sin^2B = cos^2B - cos^2A$, which involves two different angles A and B ?	$Prove\ that\ \ sin^2A - sin^2B = cos^2B - cos^2A$ LHS= $sin^2A - sin^2B$ = $1 - cos^2A - (1 - cos^2B)$ = $1 - cos^2A - 1 + cos^2B$ = $cos^2B - cos^2A$
4.	As we have algebraic equation, do we have trigonometric equation?	Yes Example (i) $sinx = cosx$ $\Longrightarrow tanx = 1 = tan45°$ $\Longrightarrow x = 45°$ Example (i) $sin^2x - 2sinx + 1 = 0$ $\Longrightarrow (sinx - 1)^2 = 0$ $\Longrightarrow sinx - 1 = 0$ $\Longrightarrow sinx - 1 = 0$ $\Longrightarrow sinx = 1 = sin90°$ $\Longrightarrow x = 90°$
5.	Is it correct to write: cos(A + B) = cosA + cosB	No. If we take A=30° and B=60° *LHS=cos(A + B)* = cos(30° + 60°) RHS= cosA + cosB = cos 90° =1 = cos30° + cos60° = $\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$ Hence $cos(A + B) \neq cosA + cosB$

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6.	Whenever we use any algebraic identity in the solution, is it important to mention the identity which we have used?	It is better to write them so that you do not go wrong while applying them.
7.	Is it necessary to learn how to use *trigonometric tables*?	Yes. It is a part of the ICSE 2021 Scope of Syllabus. Its application is there in Heights and Distances.
8.	If $tan\,A + \sin A = m$ and $tan\,A - sin\,A = n$, prove that $m^2 - n^2 = 4\sqrt{mn}$	Here $m^2 = (tan\,A + sin\,A)^2 = tan^2A + sin^2A + 2\,tan\,A\,sin\,A$ and $n^2 = (tan\,A - sin\,A)^2 = tan^2A + sin^2A - 2\,tan\,A\,sin\,A$ $\Rightarrow \quad m^2 - n^2 = 4\,tan\,A\,sin\,A = 4.\frac{sin\,A}{cos\,A} \times sin\,A = 4\frac{sin^2A}{cos\,A}$ and $mn = (tan\,A + \sin A\,)(tan\,A - sin\,A)$ $= tan^2A - sin^2A = \frac{sin^2A}{cos^2A} - sin^2A = \frac{sin^2A - sin^2A cos^2A}{cos^2A} = \frac{sin^2A(1-cos^2A)}{cos^2A} = \frac{sin^4A}{cos^2A}$ Therefore $\sqrt{mn} = \sqrt{\frac{sin^4A}{cos^2A}} = \frac{sin^2A}{cos\,A}$ $\Rightarrow\ 4\sqrt{mn} = 4\frac{sin^2A}{cos\,A}$ $m^2 - n^2 = 4\sqrt{mn}$ $= 4\frac{sin^2A}{cos\,A}$ Hence proved.
9.	Prove: $\frac{2}{1-2cos^2A} = 2\frac{sec^2A}{tan^2A-1}$	$LHS = \frac{2}{1-\frac{2}{sec^2A}} = \frac{2}{\frac{sec^2A-2}{sec^2A}}$ $= \frac{2sec^2A}{(1+tan^2A)-2}$ $= \frac{2sec^2A}{1+tan^2A-2} = \frac{2sec^2A}{tan^2A-1}$ $= RHS$
10.	In 2021 Mathematics syllabus are angles other than standard angles are included?	Yes

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11.	Prove that (Sec A-Cosec A) (1+tan A +cot A) =tan A . Sec A - cot A. Cosec A Can we prove both sides to a common answer, and not a direct proof?	Yes, in the following way: $LHS = \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$ $= \left(\frac{sin\ A - cos\ A}{sin\ A\ cos\ A}\right) \times \left(\frac{cos\ A.sin\ A + sin^2 A + cos^2 A}{cos\ A.sin\ A}\right)$ $= \frac{(sin\ A - cos\ A)(sin^2 A + sin\ A\ cos\ A + cos^2 A)}{sin^2 A.cos^2 A} = \frac{sin^3 A - cos^3 A}{sin^2 A.cos^2 A}$ $RHS = \frac{sin\ A}{cos\ A}.\frac{1}{cos\ A} - \frac{cos\ A}{sin\ A}.\frac{1}{sin\ A}$ $= \frac{sin\ A}{cos^2 A} - \frac{cos\ A}{sin^2 A} = \frac{sin^3 A - cos^3 A}{sin^2 A.cos^2 A}$ Hence LHS=RHS
12.	Prove that: $\frac{1 + tan\ A}{sin\ A} + \frac{1 + cot\ A}{cos\ A} = 2(sec\ A + cosec\ A)$	$\boldsymbol{LHS = \frac{1 + tan\ A}{sin\ A} + \frac{1 + cot\ A}{cos\ A}}$ $\boldsymbol{= \left(\frac{1}{sin\ A} + \frac{\frac{sin\ A}{cos\ A}}{sin\ A}\right) + \left(\frac{1}{cos\ A} + \frac{\frac{cos\ A}{sin\ A}}{cos\ A}\right)}$ $\boldsymbol{= \left(cosec\ A + \frac{sin\ A}{cos\ A} \times \frac{1}{sin\ A}\right) + \left(sec\ A + \frac{cos\ A}{sin\ A} \times \frac{1}{cos\ A}\right)}$ $\boldsymbol{= (cosec\ A + sec\ A) + (sec\ A + cosec\ A)}$ $\boldsymbol{= 2(sec\ A + cosec\ A)}$ $\boldsymbol{= RHS}$
13.	Can we solve the trigonometric identities using right angled triangle and applying the relation among perpendicular, base, hypotenuse?	Yes, you may but for complex sums this method may become complicated. It is more convenient to prove by using other standard identities.

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14.	Prove that $\frac{1-sin\ A}{1+sin\ A} = (sec\ A - tan\ A)^2$	$LHS = \frac{1 - sin\ A}{1 + sin\ A} = \frac{1 - sin\ A}{1 + sin\ A} \times \frac{1 - sin\ A}{1 - sin\ A}$ $= \frac{(1-sin\ A)^2}{(1)^2-(sin\ A)^2}$ $= \frac{(1-sin\ A)^2}{1-sin^2A}$ $= \frac{(1-sin\ A)^2}{cos^2A}$ $= \left(\frac{1-sin\ A}{cos\ A}\right)^2$ $= \left(\frac{1}{cos\ A} - \frac{sin\ A}{cos\ A}\right)^2$ $= (sec\ A - tan\ A)^2$ $= RHS$
15.	In board examination do we get marks for every step?	Marks are given for necessary correct relevant steps.
16.	Prove that $2(sin^6x + cos^6x) - 3(sin^4x + cos^4x) + 1 = 0$	$LHS = 2(sin^6x + cos^6x) - 3(sin^4x + cos^4x) + 1$ $= 2\{(sin^2A)^3 + (cos^2A)^3\} - 3\{(sin^2A)^2 + (cos^2A)^2\} + 1$ $= 2\{(sin^2A + cos^2A)^3 - 3sin^2A.cos^2A(sin^2A + cos^2A)\}$ $-3\{(sin^2A + cos^2A)^2 - 2sin^2Acos^2A\} + 1$ $= 2(1 - 3sin^2A.cos^2A) - 3(1 - 2sin^2A.cos^2A) + 1$ $= 2 - 6sin^2A.cos^2A - 3 + 6\ sin^2A.cos^2A + 1$ $= 0 = RHS$
17.	Is it necessary to write the basic trigonometric identities which we are using, beside the sum?	The sum will not be marked wrong if you do not write the standard identities used.