YEAR 2020 EXAMINATION

ISC

Analysis of Pupil Performance

MATHEMATICS

Research Development and Consultancy Division
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New Delhi

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FOREWORD

This document of the Analysis of Pupils' Performance at the ISC Year 12 and ICSE Year 10 Examination is one of its kind. It has grown and evolved over the years to provide feedback to schools in terms of the strengths and weaknesses of the candidates in handling the examinations.

We commend the work of Mrs. Shilpi Gupta (Deputy Head) and the Research Development and Consultancy Division (RDCD) of the Council who have painstakingly prepared this analysis. We are grateful to the examiners who have contributed through their comments on the performance of the candidates under examination as well as for their suggestions to teachers and students for the effective transaction of the syllabus.

We hope the schools will find this document useful. We invite comments from schools on its utility and quality.

Gerry Arathoon
Chief Executive & Secretary

November 2020

PREFACE

The CISCE has been involved in the preparation of the ICSE and ISC Analysis of Pupil Performance documents since the year 1994. Over these years, these documents have facilitated the teaching-learning process by providing subject/ paper wise feedback to teachers regarding performance of students at the ICSE and ISC Examinations. With the aim of ensuring wider accessibility to all stakeholders, from the year 2014, the ICSE and the ISC documents have been made available on the CISCE website www.cisce.org.

The documents for the ICSE and ISC Examination Year 2020 include a detailed qualitative analysis of the performance of students in different subjects. The purpose of this analysis is to provide insights into how candidates have performed in individual questions set in the question paper. This section is based on inputs provided by examiners from examination centers across the country. It comprises of question wise feedback on the performance of candidates in the form of *Comments of Examiners* on the common errors made by candidates along with *Suggestions for Teachers* to rectify/ reduce these errors. The *Marking Scheme* for each question has also been provided to help teachers understand the criteria used for marking. Topics in the question paper that were generally found to be difficult or confusing by candidates, have also been listed down, along with general suggestions for candidates on how to prepare for the examination/ perform better in the examination.

The Analysis of Pupil Performance document for ICSE for the Examination Year 2020 covers the following subjects/papers: English (English Language, Literature in English), History and Civics, Mathematics, Physics, Chemistry, Commercial Studies and Environmental Science.

Subjects covered in the ISC Analysis of Pupil Performance document for the Year 2020 include English (English Language and Literature in English), Hindi, Physics, Chemistry, Mathematics, Computer Science, History, Political Science, Economics, Commerce, Accounts, and Environmental Science.

I would like to acknowledge the contribution of all the ICSE and the ISC examiners who have been an integral part of this exercise, whose valuable inputs have helped put this document together.

I would also like to thank the RDCD team of Dr. M.K. Gandhi, Dr. Manika Sharma, Mrs. Roshni George and Ms. Mansi Guleria, who have done a commendable job in preparing this document.

We hope that this document will enable teachers to guide their students more effectively and comprehensively so that students prepare for the ICSE/ ISC Examinations, with a better understanding of what is required from them.

Shilpi Gupta
November 2020 ***Deputy Head - RDCD***

CONTENTS

SECTION A (80 Marks)

Question 1

[10×2]

(i) Determine whether the binary operation $*$ on R defined by $a * b = |a-b|$ is commutative. Also, find the value of $(-3)*2$.

(ii) Prove that:
$\tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 3) = 11.$

(iii) Without expanding at any stage, find the value of the determinant:

$$\Delta = \begin{vmatrix} 20 & a & b+c \\ 20 & b & a+c \\ 20 & c & a+b \end{vmatrix}$$

(iv) If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, find x.

(v) Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = 3axy$

(vi) The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long?

(vii) Evaluate: $\int_4^5 |x-5|\,dx$

(viii) Form a differential equation of the family of the curves $y^2 = 4ax$.

(ix) A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white?

(x) Let A and B be two events such that

$$P(A) = \frac{1}{2}, P(B) = p \quad \text{and} \quad P(A \cup B) = \frac{3}{5}$$

find ‘p’ if A and B are independent events.

Comments of Examiners	Suggestions for Teachers
(i) Many candidates failed to prove the given binary operation. (ii) Several candidates made errors while applying properties of inverse trigonometric functions, converting one inverse trigonometric function into another equivalent inverse trigonometric function and in simplification. (iii) In some cases, candidates expanded the determinant to solve it, though it was clearly mentioned in the question - without expanding at any stage, find the value of the determinant. (iv) A few candidates made errors while finding the product of a matrix by a matrix. (v) Errors were made by some candidates while finding the derivative of implicit function. They also made errors in simplification. (vi) A few candidates made mistake in writing the final answer in the correct form. (vii) Many candidates were unable to identify the correct intervals of absolute function for the given limits. (viii) Most of the candidates differentiated the function correctly but did not eliminate the arbitrary constant. Some of them used the second-order derivative method for eliminating constant 'a'. (ix) A number of candidates made errors while finding the probability of drawing four balls one by one with replacement. This was due to lack of knowledge of the basic idea of probability of drawing balls with or without replacement. (x) Several candidates did not find the probability of mutually exclusive events and independent events by applying the concept of independent events.	▪ *Explain the concept of modulus function and properties of binary operations like closure, commutative, identity and inverse.* ▪ *Give adequate practice in comprehension of well-defined mathematical operations.* ▪ *Explain and illustrate conversion formulae for all inverse trigonometric functions with the help of diagrams by using trigonometric ratios and Pythagoras theorem.* ▪ *Clarify the difference between expanding and not expanding the determinant at any stage. Give extensive practice in questions based on the properties of determinants.* ▪ *Explain the concept of the product of two matrices with suitable examples.* ▪ *Give frequent practice in different types of functions to help students to identify and distinguish between different types of functions.* ▪ *Explain the concept of differential coefficient as a rate measurer. Lay emphasis on the rate of change of connected variables.* ▪ *Clarify the concept of absolute function and the methods of identifying intervals where the function satisfies the given limits.* ▪ *Emphasise on framing the differential equation by differentiating and eliminating the arbitrary constants in the process of mathematical simplification.* ▪ *Clarify the concept of probability with and without replacement.* ▪ *Emphasise on mutually exclusive events and independent events.*

MARKING SCHEME

Question 1

(i)	a*b=$\lvert a\text{-}b \rvert$ b*a=$\lvert b\text{-}a\rvert = \lvert a\text{-}b\rvert$ $\Rightarrow$ a*b = b*a , hence commutative $(-3)*2=\lvert -3-2\rvert=\lvert -5\rvert=5$ =|5|
(ii)	$\tan^2(\sec^{-1}2)+\cot^2\ (\text{cosec}^{-1}3)$ $\tan^2\left(\tan^{-1}\sqrt{3}\right)+\cot^2\cot^{-1}\sqrt{8}$ $\left(\sqrt{3}\right)^2+\left(\sqrt{8}\right)^2=3+8=11$ **Or** $\tan^2(\sec^{-1}2)+\cot^2(\text{cosec}^{-1}3)$ $\sec^2(\sec^{-1}2)-1+\text{cosec}^2(\text{cosec}^{-1}3)-1$ $=4-1+9-1$ $=11$
(iii)	$\Delta=\begin{vmatrix}20 & a & b+c\\ 20 & b & a+c\\ 20 & c & a+b\end{vmatrix}$ $C_3\Rightarrow C_3+C_2\begin{vmatrix}20 & a & b+c+a\\ 20 & b & a+c+b\\ 20 & c & a+b+c\end{vmatrix}$ $20(a+b+c)\begin{pmatrix}1 & a & 1\\ 1 & b & 1\\ 1 & c & 1\end{pmatrix}=20(a+b+c)\times 0=0$
(iv)	$\begin{pmatrix}2 & 3\\ 5 & 7\end{pmatrix}\begin{pmatrix}1 & -3\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$ $\begin{pmatrix}-4 & 6\\ -9 & 13\end{pmatrix}=\begin{pmatrix}-4 & 6\\ -9 & x\end{pmatrix}$ $x=13$
(v)	$x^3+y^3=3axy$ $3x^2+3y^2\dfrac{dy}{dx}=3a(x\dfrac{dy}{dx}+y)$

	$3x^2+3y^2\frac{dy}{dx}=3ax\frac{dy}{dx}+3ay$ $\frac{dy}{dx}\left(3y^2-3ax\right)=3ay-3x^2$ $\frac{dy}{dx}=\frac{3ay-3x^2}{3y^2-3ax}=\frac{ay-x^2}{y^2-ax}$
(vi)	Let the edge of the cube be $a, \frac{da}{dt}=10cm/s$ $V=a^3$ $\frac{dv}{dt}=3a^2\frac{da}{dt}$ (differentiating with respect to t) $=3\times5\times5\times10=750cm^3/s$
(vii)	$-\int_4^5(x-5)dx$ $-\left[\frac{x^2}{2}-5x\right]_4^5$ $-\left[\left(\frac{25}{2}-25\right)-\left(8-20\right)\right]$ $-\left(\frac{-25}{2}+12\right)$ $\frac{1}{2}$
(viii)	$y^2=4ax$ $2y\frac{dy}{dx}=4a$ $2y\frac{dy}{dx}=\frac{y^2}{x}$ $2x\frac{dy}{dx}=y$
(ix)	$\begin{vmatrix}5W\\7R\\4B\end{vmatrix}$ None is white Identifying the probability of drawing not a white ball in 1st attempt = $\frac{11}{16}$ $\frac{11}{16}\times\frac{11}{16}\times\frac{11}{16}\times\frac{11}{16}=\left(\frac{11}{16}\right)^4$

(x)	$P(A)=\frac{1}{2}$, $P(B)=p$, $P\left(A\cup B\right)=\frac{3}{5}$ A and B are independent. So, $P(A\cap B)=\frac{1}{2}P$ Using $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $\frac{3}{5}=\frac{1}{2}+P-\frac{1}{2}P$ $\frac{3}{5}=\frac{1}{2}+\frac{1}{2}P$ $\frac{1}{2}P=\frac{1}{10}\Rightarrow P=\frac{1}{5}$ **Alternatively**: $P(A')=1-\frac{1}{2}=\frac{1}{2}$ and $P(B')=1-p$ $P(A\cup B)=1-P(A').P(B')$ $\frac{3}{5}=1-\frac{1}{2}(1-p)\Rightarrow p=\frac{1}{5}$

Question 2

[4]

If the function f: R→R be defined as $f(x)=\frac{3x+4}{5x-7}, \left(x\neq 7/5\right)$ and

g: R→R be defined as $g(x)=\frac{7x+4}{5x-3}, \left(x\neq 3/5\right)$

show that $(g\,o\,f)(x)=(f\,o\,g)(x)$.

Comments of Examiners	Suggestions for Teachers
Most of the candidates solved this question correctly. However, in some cases, the correct concept was applied, but the simplification was not done correctly.	▪ *Adequate practice needs to be given on fundamental operations in algebraic expressions and mathematical simplifications.*

MARKING SCHEME

Question 2

$f(x)=\frac{3x+4}{5x-7}$, $g(x)=\frac{7x+4}{5x-3}$

$gof(x)=g\left[\frac{3x+4}{5x-7}\right]$

$$gof(x) = g\left[\frac{3x+4}{5x-7}\right] = \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3}$$

$$\frac{21x + \cancel{28} + 20x - \cancel{28}}{\cancel{15x} + 20 - \cancel{15x} + 21} = \frac{41x}{41} = x$$

$$fog(x) = f\left[\frac{7x+4}{5x-3}\right]$$

$$fog(x) = f\left(\frac{7x+4}{5x-3}\right) = \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7}$$

$$= \frac{21x + \cancel{12} + 20x - \cancel{12}}{\cancel{35x} + 20 - \cancel{35x} + 21} = \frac{41x}{41} = x$$

Hence, $(gof)(x) = fog(x)$

Question 3

[4]

(a) If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then prove that

$$9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$$

OR

(b) Evaluate: $\cos\left(2\cos^{-1}x + \sin^{-1}x\right)$ at $x = \frac{1}{5}$.

Comments of Examiners	Suggestions for Teachers
Maximum number of candidates applied the formula of inverse trigonometric functions correctly but could not simplify the same.	▪ *Explain the formula of all inverse trigonometric functions for sum / difference for two or more terms and give adequate practice in solving questions to get the required result.*

MARKING SCHEME

Question 3

(a)	$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$ $\cos^{-1}\left(\frac{xy}{6} - \sqrt{\frac{4-x^2}{4}}\sqrt{\frac{9-y^2}{9}}\right) = \theta$ $\frac{xy}{6} - \frac{\sqrt{4-x^2}\sqrt{9-y^2}}{6} = \cos\theta$ $xy - 6\cos\theta = \sqrt{4-x^2}\sqrt{9-y^2}$ $\cancel{x^2y^2} + 36\cos^2\theta - 12xy\cos\theta = 36 - 4y^2 - 9x^2 + \cancel{x^2y^2}$ $9x^2 - 12xy\cos\theta + 4y^2 = 36 - 36\cos^2\theta$ $9x^2 - 12xy\cos\theta + 4y^2 = 36(1-\cos^2\theta)$ $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$
OR	
(b)	$\cos(2\cos^{-1}x + \sin^{-1}x)$ $\qquad$ $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$ $= \cos(\cos^{-1}x + \frac{\pi}{2})$ $= -\sin(\cos^{-1}x)$ $= -\sin(\sin^{-1}\sqrt{1-x^2}) = -\sqrt{1-x^2}$ At $x = \frac{1}{5}$, the value is $-\sqrt{\frac{24}{25}}$

Question 4

[4]

Using properties of determinants, show that

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2 + px - 2q^2)$$

Comments of Examiners	Suggestions for Teachers
Only some candidates attempted to solve this question by expanding the determinant without applying the properties. A few candidates applied the properties correctly but made mathematical simplification errors.	▪ *Give students clear understanding of the properties of determinants with the help of examples to avoid mathematical simplification errors.*

MARKING SCHEME

Question 4

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$$

$$C_1 \to C_1 - C_2 \begin{vmatrix} x-p & p & q \\ -(x-p) & x & q \\ 0 & q & x \end{vmatrix}$$ Applying any one property of determinants

$$(x-p)\begin{vmatrix} 1 & p & q \\ -1 & x & q \\ 0 & q & x \end{vmatrix}$$ Applying any other property of determinants

$R_2 \to R_2 + R_1$

$$(x-p)\begin{vmatrix} 1 & p & q \\ 0 & p+x & 2q \\ 0 & q & x \end{vmatrix}$$ Applying any property of determinants once again

$$(x-p)\begin{vmatrix} p+x & 2q \\ q & x \end{vmatrix}$$

$$(x-p)\left[px + x^2 - 2q^2 \right]$$

$$(x-p)\left[x^2 + px - 2q^2 \right]$$

Question 5

[4]

Verify Rolle's theorem for the function, $f(x) = -1 + \cos x$ in the interval $[0, 2\pi]$

Comments of Examiners	Suggestions for Teachers
Majority of the candidates committed errors while identifying open and closed intervals in the process of applying the properties of mean value theorems. They also made errors while finding the value of 'x' in the open interval when f '(x) = 0.	▪ *Discuss open and closed intervals and their significance with the help of examples.* ▪ *Clarify to the students that continuity is in the closed interval [ ] and differentiability is in the open interval ( ).*

MARKING SCHEME

Question 5

$f(x) = -1 + \cos x$ on $[0, 2\pi]$

i) $f(x)$ is continuous on $[0, 2\pi]$

ii) $f^1(x) = -\sin x$ exists in $(0, 2\pi)$

iii) $f(0) = -1 + 1 = 0$

$f(2\pi) = -1 + 1 = 0 \qquad f(0) = f(2\pi)$

$f^1(c) = 0 \Rightarrow$

$\sin c = 0 \Rightarrow c = 0, \pi, 2\pi, ...$

where $\pi \in (0, 2\pi)$

Identifying the value of $c = \pi$

Hence Rolle's theorem is verified.

Question 6

[4]

If $y = e^{m\sin^{-1}x}$, prove that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

Comments of Examiners	Suggestions for Teachers
Most of the candidates made errors while differentiating second time and further simplification.	▪ *Explain successive differentiation and their applications by solving numerable examples in the class.*

- *Practice needs to be given to students on how to solve a variety of problems based on successive differentiation.*

MARKING SCHEME

Question 6

$y = e^{m\sin^{-1}x}$

$$\frac{dy}{dx} = \frac{me^{m\sin^{-1}x}}{\sqrt{1-x^2}} = \frac{my}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = my$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

$$(1-x^2)\cdot\frac{2dy}{dx}\cdot\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = 2m^2y\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

Question 7

[4]

(a) The equation of tangent at (2, 3) on the curve $y^2 = px^3 + q$ is $y = 4x - 7$. Find the values of 'p' and 'q'.

OR

(b) Using L' Hospital's rule, evaluate:

$$\lim_{x\to 0}\frac{xe^x - \log(1+x)}{x^2}$$

Comments of Examiners	Suggestions for Teachers
(a) Some candidates made mathematical simplification errors while finding the values of 'p' and 'q', though the application of the concept was correct. (b) Only a few candidates made mistakes while applying L' Hospital's rule the second time. Simplification errors were once again made by some candidates.	▪ *Explain geometrical interpretation of derivative of a function and the concept of tangent and normal to the students thoroughly.* ▪ *Clarify to the students the condition of perpendicularity and parallelism with the help of different examples.* ▪ *Teach the concept and application of L' Hospital's rule clearly. Ample practice on these problems needs to be given in class.*

MARKING SCHEME

Question 7

(a)	Curve $y^2 = px^3 + q$	
	tangent $y = 4x - 7$ at (2,3), $\Rightarrow \frac{dy}{dx} = 4$	Finding slope of tangent
	$2y\frac{dy}{dx} = 3px^2$	Differentiating the curve
	$2\times3\times4 = 3p\times4$	
	$p = 2$	
	$9 = 8p + q$ as (2,3) lies on the curve	
	$q = 9 - 16 = -7$	
	Alternatively: If the candidate proves that $y = 4x - 7$ is not tangent to the curve at the point (2,3), it was accepted.	
	OR	
(b)	$\lim_{x\to0}\frac{xe^x - \log(1+x)}{x^2} \quad \left(\frac{0}{0}\right)$	
	$\lim_{x\to0}\frac{e^x + xe^x - \frac{1}{1+x}}{2x} \quad \left(\frac{0}{0}\right)$	
	Differentiating numerator and denominator	
	$\lim_{x\to0}\frac{2e^x + xe^x + \frac{1}{(1+x)^2}}{2}$	
	Differentiating once again numerator and denominator	
	$\frac{2+0+1}{2} = \frac{3}{2}$	

Question 8

[4]

(a) Evaluate: $\int \frac{dx}{\sqrt{5x-4x^2}}$

OR

(b) Evaluate: $\int \sin^3 x \ \cos^4 x \, dx$

Comments of Examiners	Suggestions for Teachers
(a) Most of the candidates made mistakes while writing the Quadratic expression in the form of difference of two perfect squares which lead to further simplification mistakes. (b) This question was not answered correctly by most candidates. They committed errors in integration by substitution and simplification of Trigonometric functions.	▪ *Encourage students to practice solving different types of special integrals.* ▪ *Explain the method of integration by substitution and give adequate practice to the students in solving trigonometric functions by integration by substitution.*

MARKING SCHEME

Question 8

(a)

$$\int \frac{dx}{\sqrt{5x-4x^2}}$$

$$\frac{1}{2}\int \frac{dx}{\sqrt{\frac{5}{4}x - x^2 + \frac{25}{64} - \frac{25}{64}}}$$

write in the form of sum / difference of two perfect squares

$$\frac{1}{2}\int \frac{dx}{\sqrt{\frac{25}{64} - \left(x^2 - \frac{5x}{4} + \frac{25}{64}\right)}}$$

$$\frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{5}{8}\right)^2 - \left(x - \frac{5}{8}\right)^2}}$$

	$=\frac{1}{2}\cdot\sin^{-1}\frac{x-\frac{5}{8}}{\frac{5}{8}}+C$ $=\frac{1}{2}\sin^{-1}\frac{8x-5}{5}+C$
OR	
(b)	$\int \sin^3 x\cos^4 x\, dx$ let $\cos x = t \Rightarrow \frac{dt}{dx} = -\sin x$ $-\int \sin^3 x\cdot t^4\cdot\frac{dt}{\sin x}$ $\int t^4 \sin^2 x\, dt$ $\int t^4(1-t^2)dt$ $\int t^6 - t^4\, dt$ $\frac{t^7}{7}-\frac{t^5}{5}+C$ $\frac{\cos^7 x}{7}-\frac{\cos^5 x}{5}+C$

Question 9

[4]

Solve the differential equation

$$\left(1+x^2\right)\frac{dy}{dx} = 4x^2 - 2xy$$

Comments of Examiners	Suggestions for Teachers
Most of the candidates solved the differential equation correctly but made errors while integrating a term at the end of the solution. A few candidates did not write the constant 'C' at the end of the solution. In a few cases, candidates were not able to identify the differential equation and incorrectly noted it as a homogeneous equation.	▪ *Give adequate practice in identifying and solving different types of differential equations.* ▪ *Clarify the importance of a constant in the solution of a differential equation.*

MARKING SCHEME

Question 9

$$\left(1+x^2\right)\frac{dy}{dx}+2xy=4x^2 \quad \text{or} \quad \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$$

$$P=\frac{2x}{1+x^2} \qquad Q=\frac{4x^2}{1+x^2}$$

$$\text{If } = \int e^{\frac{2x}{1+x^2}dx} = (1+x^2)$$

$$\Rightarrow y(1+x^2)=\int\frac{4x^2}{(1+x^2)}\cdot(1+x^2)dx+C$$

$$y(1+x^2)=\frac{4x^3}{3}+C$$

Alternate solution:

$$\int\left[(1+x^2)\frac{dy}{dx}+2xy\right]dx=\int 4x^2dx+c$$

$$(1+x^2)y=4x^3+C$$

Question 10

[4]

Three persons A, B and C shoot to hit a target. Their probabilities of hitting the target are $\frac{5}{6}$, $\frac{4}{5}$ and $\frac{3}{4}$ respectively. Find the probability that:

(i) Exactly two persons hit the target.

(ii) At least one person hits the target.

Comments of Examiners	Suggestions for teachers
Many candidates could not find the probabilities correctly under the given conditions.	▪ *Explain the concept related to the independent events as well as discuss the condition of at least and at most with students with the help of examples.*

MARKING SCHEME	
Question 10	
$P(A)=\frac{5}{6}, P(B)=\frac{4}{5}, P(C)=\frac{3}{4}$ $P(\overline{A})=\frac{1}{6}, P(\overline{B})=\frac{1}{5}, P(\overline{C})=\frac{1}{4}$	
(i)	Exactly two persons hit the target: $P(A)\,P(B)\,P(\overline{C})+P(\overline{A})\,P(B)\,P(C)+P(A)\,P(\overline{B})\,P(C)$ $\left(\frac{5}{6}\times\frac{4}{5}\times\frac{1}{4}\right)+\left(\frac{1}{6}\times\frac{4}{5}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{5}\times\frac{3}{4}\right)$ $\frac{20+12+15}{120}=\frac{47}{120}$
(ii)	At least one person hits the target: $1-\left(\frac{1}{6}\times\frac{1}{4}\times\frac{1}{5}\right)=\frac{119}{120}$

Question 11

[6]

Solve the following system of linear equations using matrices:

$$x-2y=10,\quad 2x-y-z=8,\quad -2y+z=7$$

Comments of Examiners	Suggestions for teachers
Some of the candidates made simplification errors in the process of finding the values of variables *x, y* and *z*. A few candidates made errors while calculating cofactors and inverse of a matrix.	▪ *Ensure that the students are given adequate practice in computing cofactors, the inverse of a given matrix and in finding the solution of a given system of equations in the matrix form.*

MARKING SCHEME

Question 11

$x-2y=10, \quad 2x-y-z=8, \quad -2y+z=7$

$$A=\begin{pmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{pmatrix} \quad X=\begin{pmatrix}x\\ y\\ z\end{pmatrix} \quad B=\begin{pmatrix}10\\ 8\\ 7\end{pmatrix}$$

$|A|=1\times-3+2\times2+0\times2=-3+4=1$ For determinant of A

$$\text{Adj }(A)=\begin{pmatrix}\begin{vmatrix}-1 & -1\\ -2 & -1\end{vmatrix} & \begin{vmatrix}-2 & 0\\ -2 & 1\end{vmatrix} & \begin{vmatrix}-2 & 0\\ -1 & -1\end{vmatrix}\\ -\begin{vmatrix}2 & -1\\ 0 & 1\end{vmatrix} & \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix}\\ \begin{vmatrix}2 & -1\\ 0 & -2\end{vmatrix} & -\begin{vmatrix}1 & -2\\ 0 & -2\end{vmatrix} & \begin{vmatrix}1 & -2\\ 2 & -1\end{vmatrix}\end{pmatrix}=\begin{pmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & -3\end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{pmatrix}$$

$X=A^{-1}B$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{pmatrix}\begin{pmatrix}10\\ 8\\ 7\end{pmatrix}=\begin{pmatrix}-30+16+14\\ -20+8+7\\ -40+16+21\end{pmatrix}=\begin{pmatrix}0\\ -5\\ -3\end{pmatrix}$$

$x=0, \quad y=-5, \quad z=-3$

Question 12

[6]

(a) Show that the radius of a closed right circular cylinder of given surface area and maximum volume is equal to half of its height.

OR

(b) Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

Comments of Examiners	Suggestions for Teachers
(a) Several candidates did not apply the correct formula for the surface area of a closed right circular cylinder to express the volume of the cylinder as a function to be maximised and get the same in terms of one variable. Some could not complete the second order derivative test for maximisation / minimisation. A few candidates found the value of the variable but could not prove that the radius of cylinder is equal to half of the height of the cylinder. (b) Many candidates made errors while equating the first derivative to zero and could not complete the solution. Some could not complete the second order derivative test for maximization / minimisation. A few candidates found the value of the variable but could not prove that the triangle is isosceles.	▪ *Revision of mensuration formulae of 2-dimensional and 3-dimensional figures / objects needs to be given to students.* ▪ *Explain the method of identifying the objective function, re-writing in terms of one variable and applying the concept of maxima/minima (1st derivative test and 2nd derivative test).* ▪ *Teach the method of forming a function in terms of one variable in such a manner that students understand it easily.*

MARKING SCHEME

Question 12

(a) Let Radius = *r*, Height = *h*

Surface area given:

$S = 2\pi r(r+h)$

or $h = \dfrac{S-2\pi r^2}{2\pi r}$

Volume (V)=$\pi r^2 h$

$V = \dfrac{\pi r^2 \cdot (S-2\pi r^2)}{2\pi r}$

or V=$\dfrac{1}{2}\left(rS-2\pi r^3\right)$

$\dfrac{dV}{dr} = \dfrac{1}{2}(S-6\pi r^2)$ differentiating Volume function

For maxima and minima $\dfrac{dV}{dr} = 0$

Or $\dfrac{1}{2}(S-6\pi r^2) = 0$

$r = \left(\frac{S}{6\pi}\right)^{\frac{1}{2}}$ or $S = 6\pi r^2$

$\frac{d^2V}{dr^2} = \frac{1}{2}(0 - 12\pi r) = -6\pi r = -ve$ (Negative value)

So, volume is maximum at $r = \left(\frac{S}{6\pi}\right)^{\frac{1}{2}}$

$$h = \frac{S - 2\pi r^2}{2\pi r} = \frac{S - 2\pi \cdot \frac{S}{6\pi}}{2\pi\left(\frac{S}{6\pi}\right)^{\frac{1}{2}}} = 2\left(\frac{S}{6\pi}\right)^{\frac{1}{2}} = 2r$$

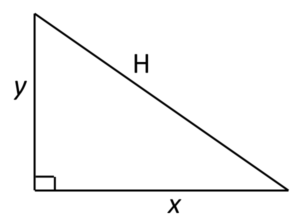
which proves that radius of the cylinder is equal to half of its height.

OR

(b) Given hypotenuse H

Let the other two sides be: $'x'$ *and* $'y'$

$H^2 = x^2 + y^2$ or $y = \sqrt{H^2 - x^2}$



Area $(A) = \frac{1}{2}xy$

$A = \frac{1}{2}x\sqrt{H^2 - x^2}$

$\frac{dA}{dn} = \frac{1}{2}\sqrt{H^2 - x^2} - \frac{1}{2}\frac{x^2}{\sqrt{H^2 - x^2}}$ differentiating Area function

For maxima and minima:

$\frac{dA}{dx} = 0$

$\Rightarrow \frac{1}{2}\sqrt{H^2 - x^2} - \frac{1}{2}\frac{x^2}{\sqrt{H^2 - x^2}} = 0$

$H^2 - x^2 = x^2$

$2x^2 = H^2$

$x = \frac{H}{\sqrt{2}}$

	$$\frac{d^2A}{dx^2}=\frac{1}{2\sqrt{H^2-x^2}}\cdot(-2x)-\frac{1}{2}\frac{\sqrt{H^2-x^2}\cdot 2x-x^2\cdot\frac{1}{2\sqrt{H^2-x^2}}(-2x)}{H^2-x^2}<0$$ Now $y=\sqrt{H^2-x^2}$ $=\sqrt{H^2-\frac{H^2}{2}}=\frac{H}{\sqrt{2}}$ which shows $x=y$ Or the triangle is Isosceles

Question 13

[6]

(a) Evaluate:

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$$

OR

(b) Evaluate: $\int \frac{2x+7}{x^2-x-2}\,dx$

Comments of Examiners	Suggestions for Teachers
(a) Most candidates attempted this problem to the extent of simplifying the inverse trigonometric function by taking correct substitution but made errors while applying the concept of integration by parts to integrate the function. (b) Majority of the candidates applied the concept of standard integrals in the right manner but made simplification errors.	▪ *Teach comprehensively the concepts of Integration by substitution and integration by parts.* ▪ *Clarify the concept of integration by partial fraction to the students thoroughly providing variety of examples.*

MARKING SCHEME

Question 13

(a)	$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

	Put $x = cos\ 2\theta, \frac{dx}{d\theta} = -2\sin 2\theta$ $= -\int \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\, 2\sin 2\theta\, d\theta$ $= -2\int \tan^{-1}\tan\theta \cdot \sin 2\theta\, d\theta$ Simplified expression be in the form where integration is possible $= -2\int \theta \sin 2\theta\, d\theta$ $= 2\left[\frac{\theta\cos 2\theta}{2} - \frac{1}{2}\int \cos 2\theta d\theta\right]$ $= 2\left[\frac{\theta\cos 2\theta}{2} - \frac{1}{2}\frac{\sin 2\theta}{2}\right] + C$ $= \theta\cos 2\theta - \frac{1}{2}\sin 2\theta + C$ $= \frac{x}{2}\cos^{-1} x - \frac{1}{2}\sqrt{1-x^2} + C$
OR	
(b)	$\int \frac{2x+7}{x^2-x-2}$ $= \int \frac{2x-1}{x^2-x-2} + \int \frac{8dx}{x^2-x-2}$ $= \log\left(x^2-x-2\right) + 8\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 - \left(2+\frac{1}{4}\right)}$ $= \log(x^2-x-2) + 8\int \frac{dx}{\left(x-\frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2}$

$$= \log\left(x^2 - x - 2\right) + 8 \times \frac{1}{2\left(\frac{2}{3}\right)} \log\left[\frac{\left(x - \frac{1}{2}\right) - \frac{3}{2}}{\left(x - \frac{1}{2}\right) + \frac{3}{2}}\right] + c$$

$$= \log\left(x^2 - x - 2\right) + \frac{8}{3} \log\left[\frac{2x - 4}{2x + 1}\right] + c$$

$$= \log\left(x^2 - x - 2\right) + \frac{8}{3} \log\left[\frac{x - 2}{x + 1}\right] + c$$

Alternate solution:

$$\int \frac{2x + 7}{(x - 2)(x + 1)} dx$$

$$\frac{2x + 7}{(x - 2)(x + 1)} = \frac{A}{x - 2} + \frac{B}{x + 1}$$

Calculate the values of A and B

A=11/3 & B=-5/3

$$= \int \frac{A}{x - 2} dx + \int \frac{B}{x + 1} dx$$

$$= \frac{11}{3} \int \frac{1}{x - 2} dx - \frac{5}{3} \int \frac{1}{x + 1} dx$$

$$= \frac{11}{3} \log(x - 2) - \frac{5}{3} \log(x + 1) + C$$

Question 14

[6]

The probability that a bulb produced in a factory will fuse after 150 days of use is 0·05. Find the probability that out of 5 such bulbs:

(i) None will fuse after 150 days of use.

(ii) Not more than one will fuse after 150 days of use.

(iii) More than one will fuse after 150 days of use.

(iv) At least one will fuse after 150 days of use.

Comments of Examiners	Suggestions for Teachers
Most candidates who attempted this question committed simplification errors while finding the probability values of success and failure. These incorrect values led the incorrect solutions. Some candidates made errors while calculating the probability in the case of bulbs *not more than one will fuse after 150 days of use.*	▪ *Ensure that students understand the concepts of probability distribution and their properties.* ▪ *Ample practice needs to be given on different types of probability distribution problems.*

MARKING SCHEME

Question 14

It is a case of binomial distribution. $p=0\cdot05,\ q=0\cdot95,\ n=5$ Binomial distribution $(q+p)^5$	
(i)	None fuse: $P(X=0)=(\cdot95)^5$
(ii)	Not more than one fuse $P(X=0)+P(X=1)=5C_o q^5+5q\,q^4\,p^1$ $(\cdot95)^5+5(\cdot95)^4(\cdot05)^1$
(iii)	More than one fuse $1-\left[(\cdot95)^5+5(\cdot95)^4(\cdot05)^1\right]$
(iv)	At least one fuses $1-(\cdot95)^5$

SECTION B (20 Marks)

Question 15

[3×2]

(a) Write a vector of magnitude of 18 units in the direction of the vector $\hat{\imath}-2\hat{\jmath}-2\hat{k}$.

(b) Find the angle between the two lines:

$$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4} \quad \text{and} \quad \frac{x-1}{5}=\frac{y+2}{2}=\frac{z-1}{-5}$$

(c) Find the equation of the plane passing through the point (2, -3, 1) and perpendicular to the line joining the points (4, 5, 0) and (1, -2, 4).

Comments of Examiners	Suggestions for Teachers
(a) Majority of the candidates answered this question correctly. However, a few candidates applied the formula of unit vector for the given vector incorrectly. (b) In some cases, candidates applied the formula for the angle between the two lines correctly but committed errors in the process of simplification. (c) Most candidates attempted this part well, barring a few who applied the correct concept but made simplification errors at the end.	▪ *Discuss the concept of unit vector and magnitude of a vector, dot, and cross product of two vectors and their properties with examples.* ▪ *Teach thoroughly the concept of a plane and direction ratios normal to the plane and finding the equation of a plane satisfying different conditions.* ▪ *Give adequate practice to the students in solving problems based on the above concepts.*

MARKING SCHEME

Question 15

(a)	Unit vector in the direction of $\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ is $\frac{\hat{\imath}-2\hat{\jmath}-2\hat{k}}{\sqrt{1+4+4}} = \frac{1}{3}(\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$ Vector of magnitude in its direction will be: $18 \times \frac{1}{3}(i - 2j - 2k) = 6(i - 2j - 2k)$
(b)	Lines are $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{5} = \frac{y+2}{2} = \frac{z-1}{-5}$ $\cos\theta = \frac{10+10-20}{\sqrt{45}\sqrt{54}} = 0$ $\Rightarrow \theta = \frac{\pi}{2}$
(c)	Passes through the point (2, -3, 1) dr's=<3, 7, -4> Equation: $3(x-2) + 7(y+3) - 4(z-1) = 0$ $3x + 7y - 4z + 19 = 0$

Question 16

[4]

(a) Prove that $\vec{a}.\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+3\vec{b}+4\vec{c}\right)\right] = [\,\vec{a}\quad\vec{b}\quad\vec{c}\,]$

OR

(b) Using vectors, find the area of the triangle whose vertices are:

A (3,−1, 2), B (1,−1,−3) and C (4,−3, 1)

Comments of Examiners	Suggestions for Teachers
(a) A few candidates made simplification errors in applying dot and cross product while expanding the left-hand side of the equation. (b) Many candidates made errors in the process of finding any two sides of the triangle and some applied incorrect formula to find the area of the triangle.	▪ *The students need to be taught comprehensively on the area of a triangle by using position vectors and applications based on the concept.* ▪ *Give ample practice in solving problems to avoid simplification errors to the maximum extent.*

MARKING SCHEME

Question 16

(a)

$\vec{a}.\left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+3\vec{b}+4\vec{c}\right)\right]$

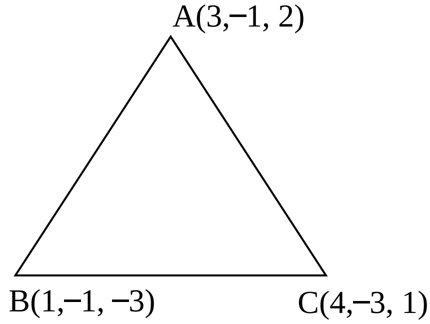
$\vec{a}.\left[(\vec{b}\times\vec{a})+(\vec{b}\times 3\vec{b})+(\vec{b}\times\overrightarrow{4c})+(\vec{c}\times\vec{a})+(\vec{c}\times 3\vec{b})+(\vec{c}\times 4\vec{c})\right]$

$\vec{a}.\left[\left(\vec{b}\times\vec{a}\right)+0+\left(\vec{b}\times\overrightarrow{4c}\right)+(\vec{c}\times\vec{a})-\left(3\vec{b}\times\vec{c}\right)+0\right]=$

[a b a] + 4 [a b c] + [a c a] – 3 [a b c] = [a b c]

OR

(b)



	$\overrightarrow{BA} = 2\hat{\imath} + 5\hat{k}$ $\overrightarrow{BC} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ $\frac{1}{2}\left\|\overrightarrow{BA}\times\overrightarrow{BC}\right\|$ Area = $=\left\|\frac{1}{2}\begin{vmatrix} i & j & k \\ 2 & 0 & 5 \\ 3 & -2 & 4 \end{vmatrix}\right\|$ $\left\|\frac{1}{2}\|10i+7j-4k\|\right\| = \frac{1}{2}\sqrt{100+49+16}$ $= \frac{1}{2}\sqrt{165}$

Question 17

[4]

(a) Find the image of the point (3,−2, 1) in the plane $3x - y + 4z = 2$

OR

(b) Determine the equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the lines:

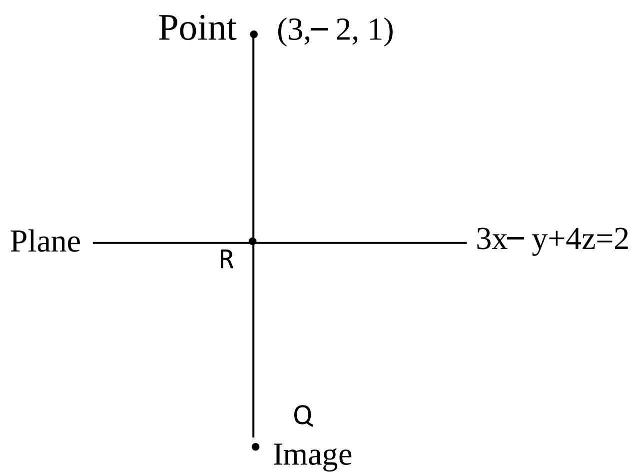
$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \quad \text{and} \quad \frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$$

Comments of Examiners	Suggestions for Teachers
(a) A number of candidates found the incorrect value of the image of the point in the given plane. A few candidates applied irrelevant method to solve the problem. (b) Most of the candidates did not apply the concept of perpendicular lines and the condition for the same correctly. Some candidates made calculation errors while applying the cross-multiplication rule to find the direction ratios.	▪ *Stress on concept building by giving adequate problems on three-dimensional geometry number for practice.* ▪ *Show students the different methods of finding the equation of a straight line by applying different types of conditions.*

MARKING SCHEME

Question 17

(a) The line PQ $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = k$ **is** perpendicular to given plane $3x - y + 4z = 2$



Let the point R be

(3k+3, −k− 2, 4k+1)

This satisfies the plane $3x - y + 4z = 2$

$$3(3k+3) - (-k-2) + 4(4k+1) = 2$$

$$9k+9+k+2+16k+4=2$$

$$26k = -13,\ k = -\frac{1}{2}$$

Point R is $\left(\frac{-3}{2}+3, \frac{1}{2}-2, \frac{-4}{2}+1\right) = \left(\frac{3}{2}, \frac{-3}{2}, -1\right)$

$$\frac{\alpha+3}{2} = \frac{3}{2}, \frac{\beta-2}{2} = \frac{-3}{2}, \frac{y+1}{2} = 1 \quad \text{image } (0,-1,-3)$$

$$\alpha=0, \beta=-1,\ y=-3$$

OR

(b) Equation of a line passing through (-1. 3, -2) is:

$$\frac{x+1}{a} = \frac{y-3}{b} = \frac{z+2}{c}$$

Perpendicular to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ *and* $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

	Gives a+2b+3c=0 -3a + 2b + 5c = 0 $\Rightarrow \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6} \Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$ or $\frac{a}{2} = \frac{b}{-7} = \frac{c}{4}$ So, the required equation of the line is: $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$

Question 18

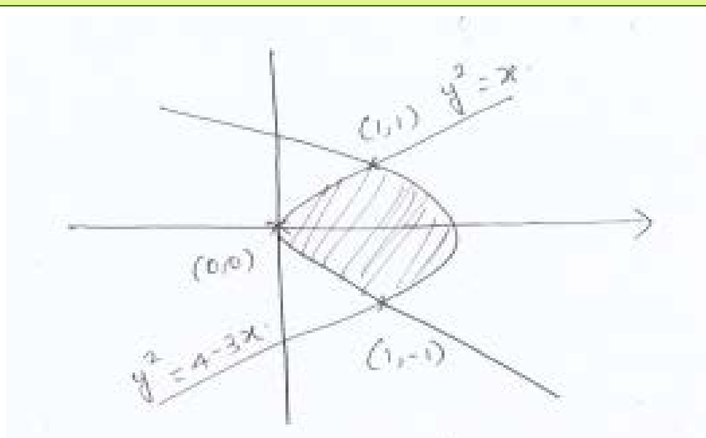
[6]

Draw a rough sketch of the curves $y^2 = x$ and $y^2 = 4 - 3x$ and find the area enclosed between them.

Comments of Examiners	Suggestions for Teachers
Only a few candidates attempted this question well. Many candidates were not able to associate the equations of the curves as a parabola and made mistakes while sketching the graph. The concept of symmetry was not used by a few candidates and hence, they could not find the common portion and limits correctly.	▪ *Give sufficient practice in sketching various types of curves and interpretation of various graphs.* ▪ *Explain the method of identifying the limits of a definite integral and the area bounded by the region for the given curves and X-axis, etc.*

MARKING SCHEME

Question 18



Points of intersection are (1,-1) and (1,1)

Area between the curves:

$$\int_{-1}^{1} x\,dy \quad - \int_{-1}^{1} x\,dy$$

$$\left(y^2=4-3x\right) \qquad \left(y^2=x\right)$$

$$\int_{-1}^{1} \frac{4-y^2}{3}dy \; - \int_{-1}^{1} y^2 dy$$

Integration of both terms with limits

$$\left(\frac{4y}{3}-\frac{y^3}{9}\right)_{-1}^{1} - \left(\frac{y^3}{3}\right)_{-1}^{1}$$

$$\left[\left(\frac{4}{3}-\frac{1}{9}\right)-\left(\frac{-4}{3}+\frac{1}{9}\right)\right]-\left[\frac{1}{3}+\frac{1}{3}\right] = \frac{16}{9} \text{sq. units}$$

SECTION C (20 Marks)

Question 19

[3×2]

(a) The selling price of a commodity is fixed at ₹ 60 and its cost function is C $(x) = 35\,x + 250$

(i) Determine the profit function.

(ii) Find the break even points.

(b) The revenue function is given by $R(x) = 100x - x^2 - x^3$. Find

(i) The demand function.

(ii) Marginal revenue function.

(c) For the lines of regression $4x - 2y = 4$ and $2x - 3y + 6 = 0$, find the mean of ‘x’ and the mean of ‘y’.

Comments of Examiners	Suggestions for Teachers
(a) This question was well attempted by most candidates. A few candidates, however made calculation errors in simplification. (b) Most of the candidates attempted this part correctly, but a few candidates made mistakes while calculating the demand function and marginal revenue function. (c) Most of the candidates solved this question by correctly identifying the mean of variables 'x' and 'y'. In some cases, candidates applied the concept of solving regression equations correctly but again made simplification errors.	▪ *Explain all technical terms and formulae of the chapter on cost function along with examples.* ▪ *Explain the concept of the mean of x and y by solving the two equations simultaneously.* ▪ *Advise students to maintain the highest level of accuracy while solving problems from this topic.*

MARKING SCHEME

Question 19

(a)	$p=60,\ C(x)=35x+250$ $R(x)=60x$	
	(i)	Profit function $P(x)=60x-35x-250$ $25x-250$
	(ii)	Break even points $60x=35x+250$ $x=10$
(b)	$R(x)=100x-x^2-x^3$	
	(i)	Demand function: $p=100-x-x^2$
	(ii)	Marginal revenue = $100-2x-3x^2$ is the marginal revenue
(c)	$4x-2y=4$ $2x-3y=-6$ On solving simultaneously, we get mean of x = 3 and mean of y = 4	

Question 20

[4]

(a) The correlation coefficient between x and y is 0.6. If the variance of x is 225, the variance of y is 400 , mean of x is 10 and mean of y is 20 , find

(i) the equations of two regression lines.

(ii) the expected value of y when $x = 2$

OR

(b) Find the regression coefficients b_{yx} , b_{xy} and correlation coefficient 'r' for the following data : (2,8), (6,8), (4,5), (7, 6), (5 ,2)

Comments of Examiners	Suggestions for Teachers
(a) Many candidates applied the incorrect formulae for calculation of b_{yx} and b_{xy}, thereby made errors in finding the regression equations which led to incorrect expected value of 'y' when x=2. (b) Many candidates wrote incorrect formulas for regression coefficients which caused errors in finding the regression coefficients hence led to the incorrect correlation coefficient.	▪ *Ensure that students learn formulae for calculating regression coefficients b_{yx} and b_{xy} correctly.* ▪ *Advise students to practice multiple problems based on regression.*

MARKING SCHEME

Question 20

(a) (i)

$$r = 0\cdot 6, \quad \sigma_x^2 = 225, \quad \bar{x} = 10, \quad \bar{y} = 20$$

$$\downarrow$$

$$\sigma_x = 15 \quad \sigma_y = 20$$

$$byx = \frac{\overset{2}{\cancel{6}}}{\cancel{10}} \times \frac{\overset{2}{\cancel{20}}}{\underset{5}{\cancel{15}}} = \frac{4}{5}$$

$$b_{xy} = byx = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{20}}} = \frac{9}{20}$$

Line y on X : $y - 20 = \frac{4}{5}(x - 10)$

Line x on Y : $x - 10 = \frac{9}{20}(y - 20)$

(ii) when $x = 2$, $y = \frac{68}{5}$

(b)

x	y	x^2	y^2	xy
2	8	4	64	16

6	8	36	64	48
4	5	16	25	20
7	6	49	36	42
5	2	25	4	10
24	29	130	193	136

$$byx = \frac{136 - \frac{24\times 29}{5}}{130 - \frac{24\times 24}{5}} \qquad b_{xy} = \frac{136 - \frac{24\times 29}{5}}{193 - \frac{29\times 29}{5}}$$

$$= \frac{680 - 696}{650 - 576} \qquad = \frac{680 - 696}{965 - 841}$$

$$= \frac{-16}{74} \qquad = \frac{-16}{124}$$

$$= \frac{-8}{37} \qquad = \frac{-4}{31}$$

$$r = -\sqrt{\frac{8}{37}\times\frac{4}{31}} = -\sqrt{\frac{32}{1147}} = -0\cdot 167$$

Question 21

[4]

(a) The marginal cost of the production of the commodity is 30 + 2*x*, it is known that fixed costs are ₹ 200, find

(i) The total cost.

(ii) The cost of increasing output from 100 to 200 units.

OR

(b) The total cost function of a firm is given by C(*x*) = $\frac{1}{3}x^3 - 5x^2 + 30x - 15$ where the selling price per unit is given as ₹ 6 . Find for what value of *x* will the profit be maximum.

Comments of Examiners	Suggestions for Teachers
(a) Several candidates integrated MC(x) to find C(x) without adding a constant to C(x). This led to an incorrect solution. Some candidates committed simplification errors in the process of applying limits to find an increase in the output cost. (b) Many candidates made errors while finding the profit function. Some made errors in the process of finding the value of '*x*' at which the profit was maximum while applying the concept of maxima and minima. In a few cases, candidates maximised the cost function in place of the profit function as well as they failed to apply the second-order derivative for maximisation.	▪ *Explain with the help of examples, the concept and interpretation of all Marginal cost / revenue functions and application of calculus in Commerce and Economics.* ▪ *The concepts of application of derivatives regarding increasing / decreasing functions and maxima / minima in the context of functions like Cost function, Marginal Cost and Average Cost, etc. needs to be taught thoroughly in class.* ▪ *Revise with students, the application of integration in Commerce and Economics supported with an appropriate number of examples.*

MARKING SCHEME

Question 21

(a)	(i)	$MC = 30 + 2x$ $C = \int 30 + 2x\,dx$ $= 30x + \dfrac{2x^2}{2} + 200$ $30x + x^2 + 200$
	(ii)	$\int_{100}^{200} (30+2x)dx = \left(30x + x^2\right)_{100}^{200}$ $\left(6000 + 40000\right) - \left(3000 + 10000\right)$ $46000 - 13000 = 33000$
OR		
(b)		$C = \dfrac{x^3}{3} - 5x^2 + 30x - 15,\ p = 6$ So, $R = 6x$ Profit function $P(x) = 6x - \dfrac{x^3}{3} + 5x^2 - 30x + 15$ $\dfrac{dp}{dx} = 6 - x^2 + 10x - 30$

	For maximum profit $\frac{dp}{dx}=0$ $6-x^2+10x-30=0$ $x^2-10x+24=0$ $(x-6)(x-4)=0$ $x=6,\ x=4$ $\frac{d^2P}{dx^2}=-2x+10$ At $x=4,\ \frac{d^2P}{dx^2}=+ve$, At $x=6,\ \frac{d^2P}{dx^2}=-ve$ So, max at $x=6$

Question 22

[6]

A company uses three machines to manufacture two types of shirts, half sleeves and full sleeves. The number of hours required per week on machine M_1 , M_2 and M_3 for one shirt of each type is given in the following table :

	M_1	**M_2**	**M_3**
Half sleeves	1	2	8/5
Full sleeves	2	1	8/5

None of the machines can be in operation for more than 40 hours per week. The profit on each half sleeve shirt is ₹ 1 and the profit on each full sleeve shirt is ₹1·50. How many of each type of shirts should be made per week to maximise the company's profit?

Comments of Examiners

Many candidates could not express the given constraints in the form of linear inequalities correctly. Several candidates did not write non-negative constraints $x \geq 0$ and $y \geq 0$.

Suggestions for Teachers

- *Revise the concept of solving linear inequalities studied in the previous class.*
- *Explain the method of identifying the constraints and forming the corresponding linear inequalities and sketching the graph of the same to identify the feasible region for the objective function.*

MARKING SCHEME

Question 22

	Quality	M_1	M_2	M_3	Profit
Half sleeves	x	1	2	$\frac{8}{5}$	1
Full sleeves	y	2	1	$\frac{8}{5}$	1·50

Maximise $z = x + 1\cdot 5y$

Under the constraints

$x + 2y \le 40, \quad 2x + y \le 40, \quad \frac{8}{5}x + \frac{8}{5}y \le 40$ Or $x + y \le 25$

$x \ge 0, \quad y \ge 0$

The corner points of the feasible region are
$A(0,20)$ $B(10,15)$ $C(15,10)$ $D(20,0)$ $O(0,0)$

At $A(0,20)$ $Z = 30$

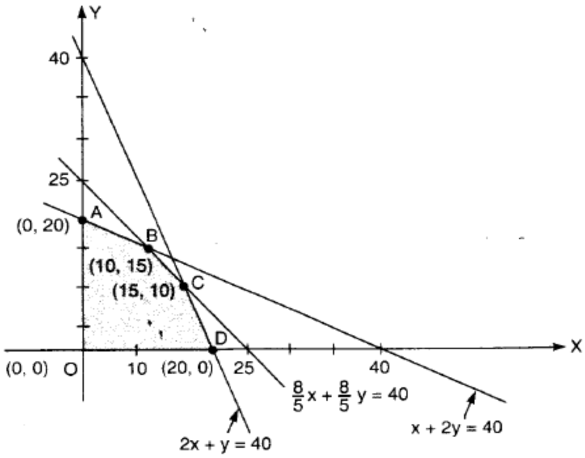
At $B(10,15)$ $Z = 32\cdot 50$

At $C(15,10)$ $Z = 30$

At $D(20,0)$ $Z = 20$

At $O(0,0) Z = 0$ Finding values of x *and* y for maximum profit

The company should make 10 half sleeve shirts and 15full sleeve shirts, each week for the maximum profit.



Note: For questions having more than one correct answer/solution, alternate correct answers/solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult/confusing by candidates

- Continuity and derivability
- Modulus function
- Applications on derivatives
- Integration by substitution and by parts
- Applications integrals
- 3D geometry
- Inverse circular functions
- Mean value theorems, open and closed interval in Mean Value theorem.
- Probability, mutually exclusive and independent events, Product and sum rule of probability, dependent and independent events.
- Identification of the types of differential equations and their framing.
- Between the regression coefficients b_{yx} and b_{xy} and regression lines y on x and x on y.
- Linear Programming Problems in identifying the constraints.
- Application of definite integrals**:** Sketching of curves, Identifying the area of the shaded region and finding the upper and lower limits from the graph.

Suggestions for Students

- Study and practice the entire syllabus keeping in view the pattern and the weightage of each chapter in the question paper. Avoid selective study.
- Stress on learning the concepts of topics like calculus and its applications as its weightage is maximum in the syllabus.
- Understand the concepts of each topic and practice an adequate number of problems on a regular basis, taking the assistance of the teacher, wherever required.
- Make a list of all formulae and revise them frequently.
- Take part in all periodic tests conducted by the school sincerely, identify the areas of weakness and try to resolve your problems.
- Revise the concepts of Class XI and integrate them with the Class XII syllabus (factorization, formulae of algebraic expressions, etc.)
- Make use of the given reading time of 15 minutes, for reading the questions carefully and highlighting various key points in the statements of the questions.
- Avoid calculation errors and increase accuracy to the highest levels while solving problems based on numerical values.
- Manage your time effectively while attempting the question paper. Practice mock/sample papers by strictly adhering to the stipulated time.