

# MATHEMATICS (860)

## CLASS XI

There will be **two** papers in the subject:

**Paper I** : Theory (3 hours) .....80 marks

**Paper II**: Project Work .....20 marks

### PAPER I (THEORY) – 80 Marks

The syllabus is divided into **three** sections A, B and C.

Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **EITHER** Section B **OR** Section C.

There will be one paper of **three** hours duration of 80 marks.

**Section A (65 Marks)**: Candidates will be required to attempt **all** questions. Internal choice will be provided in two questions of two marks, two questions of four marks and two questions of six marks each.

**Section B/ Section C (15 Marks)**: Candidates will be required to attempt **all** questions **EITHER** from Section B or Section C. Internal choice will be provided in one question of two marks and one question of four marks.

### DISTRIBUTION OF MARKS FOR THE THEORY PAPER

| S.No.                      | UNIT                                       | TOTAL WEIGHTAGE |
|----------------------------|--------------------------------------------|-----------------|
| <b>SECTION A: 65 Marks</b> |                                            |                 |
| 1.                         | Sets and Functions                         | 20 Marks        |
| 2.                         | Algebra                                    | 24 Marks        |
| 3.                         | Coordinate Geometry                        | 8 Marks         |
| 4.                         | Calculus                                   | 6 Marks         |
| 5.                         | Statistics & Probability                   | 7 Marks         |
| <b>SECTION B: 15 marks</b> |                                            |                 |
| 6.                         | Conic Section                              | 7 Marks         |
| 7.                         | Introduction to Three-Dimensional Geometry | 5 Marks         |
| 8.                         | Mathematical Reasoning                     | 3 Marks         |
| <b>OR</b>                  |                                            |                 |
| <b>SECTION C: 15 Marks</b> |                                            |                 |
| 9.                         | Statistics                                 | 5 Marks         |
| 10.                        | Correlation Analysis                       | 4 Marks         |
| 11.                        | Index Numbers & Moving Averages            | 6 Marks         |
| <b>TOTAL</b>               |                                            | <b>80 Marks</b> |

## SECTION A

### 1. Sets and Functions

#### (i) Sets

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Practical problems on union and intersection of two and three sets. Difference of sets. Complement of a set. Properties of Complement of Sets.

#### (ii) Relations & Functions

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto  $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ ). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Function as a type of mapping, types of functions (one to one, many to one, onto, into) domain, co-domain and range of a function.

- *Sets: Self-explanatory.*
- **Basic concepts of Relations and Functions**
  - *Ordered pairs, sets of ordered pairs.*
  - *Cartesian Product (Cross) of two sets, cardinal number of a cross product.*
  - Relations as:*
    - *an association between two sets.*
    - *a subset of a Cross Product.*
    - *Domain, Range and Co-domain of a Relation.*
    - *Functions:*
      - *As special relations, concept of writing "y is a function of x" as  $y = f(x)$ .*
      - *Introduction of Types: one to one, many to one, into, onto.*
      - *Domain and range of a function.*

#### (iii) Trigonometry

Signs of trigonometric functions. Expressing  $\sin(x \pm y)$  and  $\cos(x \pm y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$  &  $\cos y$  and their simple

applications. Deducing the identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

Identities related to  $\sin 2x$ ,  $\cos 2x$ ,  $\tan 2x$ ,  $\sin 3x$ ,  $\cos 3x$  and  $\tan 3x$ .

#### • **Trigonometric Functions**

- *Relationship between trigonometric functions.*
- *Proving simple identities.*
- *Signs of trigonometric functions.*
- *Domain and range of the trigonometric functions.*
- *Trigonometric functions of all angles.*
- *Periods of trigonometric functions.*

#### • **Compound and multiple angles**

- *Addition and subtraction formula:  $\sin(A \pm B)$ ;  $\cos(A \pm B)$ ;  $\tan(A \pm B)$ ;  $\tan(A + B + C)$  etc., Double angle, triple angle, half angle and one third angle formula as special cases.*
- *Sum and differences as products*  

$$\sin C + \sin D = 2 \sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right), \text{ etc.}$$
- *Product to sum or difference i.e.  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$  etc.*

### 2. Algebra

#### (i) Principle of Mathematical Induction

Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least

inductive subset of real numbers. The principle of mathematical induction and simple applications.

Using induction to prove various summations, divisibility.

(ii) Complex Numbers

Introduction of complex numbers and their representation, Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Square root of a complex number. Cube root of unity.

- Conjugate, modulus and argument of complex numbers and their properties.
- Sum, difference, product and quotient of two complex numbers additive and multiplicative inverse of a complex number.
- Square root of a complex number.
- Cube roots of unity and their properties.

(iii) Quadratic Equations

Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients).

- Use of the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

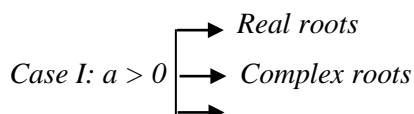
In solving quadratic equations.

- Nature of roots
  - Product and sum of roots.
  - Roots are rational, irrational, equal, reciprocal, one square of the other.
  - Complex roots.
  - Framing quadratic equations with given roots.

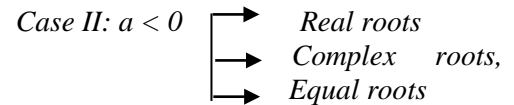
**NOTE:** Questions on equations having common roots are to be covered.

- **Quadratic Functions.**

Given  $\alpha, \beta$  as roots then find the equation whose roots are of the form  $\alpha^3, \beta^3$ , etc.



Equal roots



Where 'a' is the coefficient of  $x^2$  in the equations of the form  $ax^2 + bx + c = 0$ .

Understanding the fact that a quadratic expression (when plotted on a graph) is a parabola.

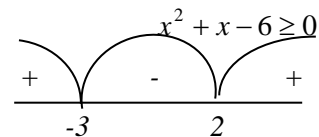
- **Sign of quadratic**

Sign when the roots are real and when they are complex.

- **Inequalities**

- Quadratic Inequalities

Using method of intervals for solving problems of the type:



A perfect square e.g.  $x^2 - 6x + 9 \geq 0$ .

(iv) Permutations and Combinations

Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of formulae for  ${}^n P_r$  and  ${}^n C_r$  and their connections, simple application.

- Factorial notation  $n!$ ,  $n! = n(n-1)!$
- Fundamental principle of counting.
- Permutations
  - ${}^n P_r$ .
  - Restricted permutation.
  - Certain things always occur together.
  - Certain things never occur.
  - Formation of numbers with digits.
  - Word building - repeated letters - No letters repeated.
  - Permutation of alike things.
  - Permutation of Repeated things.

- **Combinations**
  - ${}^nC_r, {}^nC_n = 1, {}^nC_0 = 1, {}^nC_r = {}^nC_{n-r}, {}^nC_x = {}^nC_y$ , then  $x + y = n$  or  $x = y$ ,  ${}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$ .
  - When all things are different.
  - When all things are not different.

(v) Binomial Theorem

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

- *Significance of Pascal's triangle.*
- *Binomial theorem (proof using induction) for positive integral powers,*

*i.e.  $(x + y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + \dots + {}^nC_ny^n$ .*

*Questions based on the above.*

(vi) Sequence and Series

Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of first  $n$  terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Formulae for the following special sums  $\sum n, \sum n^2, \sum n^3$ .

- **Arithmetic Progression (A.P.)**
  - $T_n = a + (n - 1)d$
  - $S_n = \frac{n}{2}\{2a + (n - 1)d\}$
  - *Arithmetic mean:  $2b = a + c$*
  - *Inserting two or more arithmetic means between any two numbers.*
  - *Three terms in A.P. :  $a - d, a, a + d$*
  - *Four terms in A.P.:  $a - 3d, a - d, a + d, a + 3d$*

- **Geometric Progression (G.P.)**
  - $T_n = ar^{n-1}, S_n = \frac{a(r^n - 1)}{r - 1}$ ,
  - $S_\infty = \frac{a}{1 - r}; |r| < 1$  *Geometric*
  - *Mean,  $b = \sqrt{ac}$*
  - *Inserting two or more Geometric Means between any two numbers.*
  - *Three terms are in G.P.  $ar, a, ar^{-1}$*
  - *Four terms are in GP  $ar^3, ar, ar^{-1}, ar^{-3}$*
  - *Special sums  $\sum n, \sum n^2, \sum n^3$*

**3. Coordinate Geometry**

(i) Straight Lines

Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Angle between two lines. Various forms of equations of a line: intercept form and normal form. General equation of a line. Distance of a point from a line.

- *Basic concepts of Points and their coordinates.*
  - *Angle between two lines.*
  - *Intercept form.*
  - *Perpendicular /normal form.*
  - *General equation of a line.*
  - *Distance of a point from a line.*
  - *Distance between parallel lines.*
  - *Equation of lines bisecting the angle between two lines.*
  - *Definition of a locus.*

(ii) Circles

- *Equations of a circle in:*
  - *Standard form.*
  - *Diameter form.*
  - *General form.*
  - *Parametric form.*
- *Given the equation of a circle, to find the centre and the radius.*
- *Finding the equation of a circle.*
  - *Given three non collinear points.*

- Given other sufficient data for example centre is  $(h, k)$  and it lies on a line and two points on the circle are given, etc.

#### 4. Calculus

##### (i) Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically.

Intuitive idea of limit. Limits of polynomials and rational functions, trigonometric functions. Definition of derivative relate it to slope of tangent of the curve, Derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

- *Limits*
  - *Notion and meaning of limits.*
  - *Fundamental theorems on limits (statement only).*
  - *Limits of algebraic and trigonometric functions.*
- *Differentiation*
  - *Meaning and geometrical interpretation of derivative.*
  - *Derivatives of simple algebraic and trigonometric functions and their formulae.*
  - *Differentiation using first principles.*
  - *Derivatives of sum/difference.*
  - *Derivatives of product of functions.*
  - *Derivatives of quotients of functions.*

#### 5. Statistics and Probability

##### (i) Statistics

Measures of dispersion: range, mean deviation, variance and standard deviation of ungrouped/grouped data.

- *Mean deviation about mean.*
- *Standard deviation - by direct method, short cut method and step deviation method.*

**NOTE:** Mean of grouped and ungrouped data are required to be covered.

##### (ii) Probability

Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories studied in earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

- *Random experiments and their outcomes.*
- *Events: sure events, impossible events, mutually exclusive and exhaustive events.*
  - *Definition of probability of an event*
  - *Laws of probability addition theorem.*

#### SECTION B

#### 6. Conic Section

Sections of a cone, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola.

- *Conics as a section of a cone.*
  - *Definition of Foci, Directrix, Latus Rectum.*
  - *$PS = ePL$  where  $P$  is a point on the conics,  $S$  is the focus,  $PL$  is the perpendicular distance of the point from the directrix.*

##### (i) Parabola

$$e = 1, y^2 = \pm 4ax, x^2 = 4ay, y^2 = -4ax, x^2 = -4ay$$

- *Rough sketch of the above.*
- *The latus rectum; quadrants they lie in; coordinates of focus and vertex; and equations of directrix and the axis.*
- *Finding equation of Parabola when Foci and directrix are given, etc.*
- *Application questions based on the above.*

(ii) Ellipse

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, e < 1, b^2 = a^2(1 - e^2)$
- Cases when  $a > b$  and  $a < b$ .
- Rough sketch of the above.
- Major axis, minor axis; latus rectum; coordinates of vertices, focus and centre; and equations of directrices and the axes.
- Finding equation of ellipse when focus and directrix are given.
- Simple and direct questions based on the above.
- Focal property i.e.  $SP + SP' = 2a$ .

(iii) Hyperbola

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, e > 1, b^2 = a^2(e^2 - 1)$
- Cases when coefficient  $y^2$  is negative and coefficient of  $x^2$  is negative.
- Rough sketch of the above.
- Focal property i.e.  $SP - S'P = 2a$ .
- Transverse and Conjugate axes; Latus rectum; coordinates of vertices, foci and centre; and equations of the directrices and the axes.
- General second-degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ 
  - Case 1: pair of straight line if  $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ,
  - Case 2:  $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ , then represents a parabola if  $h^2 = ab$ , ellipse if  $h^2 < ab$ , and hyperbola if  $h^2 > ab$ .

## 7. Introduction to three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

- As an extension of 2-D
- Distance formula.
- Section and midpoint form

## 8. Mathematical Reasoning

Mathematically acceptable statements. Connecting words/ phrases - consolidating the

understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "and", "or", "there exists" and their use through variety of examples related to the Mathematics and real life. Difference between contradiction, converse and contrapositive.

Self-explanatory.

## SECTION C

### 9. Statistics

- Combined mean and standard deviation.
- The Median and Quartiles.

### 10. Correlation Analysis

- Definition and meaning of covariance.
- Coefficient of Correlation by Karl Pearson. If  $x - \bar{x}, y - \bar{y}$  are small non-fractional numbers, we use

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2} \sqrt{\sum(y - \bar{y})^2}}$$

If  $x$  and  $y$  are small numbers, we use

$$r = \frac{\sum xy - \frac{1}{N} \sum x \sum y}{\sqrt{\sum x^2 - \frac{1}{N} (\sum x)^2} \sqrt{\sum y^2 - \frac{1}{N} (\sum y)^2}}$$

Otherwise, we use assumed means

$A$  and  $B$ , where  $u = x - A, v = y - B$

$$r = \frac{\sum uv - \frac{1}{N} (\sum u)(\sum v)}{\sqrt{\sum u^2 - \frac{1}{N} (\sum u)^2} \sqrt{\sum v^2 - \frac{1}{N} (\sum v)^2}}$$

### 11. Index Numbers and Moving Averages

(i) Index Numbers

- Price index or price relative.
- Simple aggregate method.
- Weighted aggregate method.
- Simple average of price relatives.
- Weighted average of price relatives (cost of living index, consumer price index).

- (ii) Moving Averages
- *Meaning and purpose of the moving averages.*
  - *Calculation of moving averages with the given periodicity and plotting them on a graph.*

## PAPER II – PROJECT WORK – 20 Marks

Candidates will be expected to have completed **two** projects, one from Section A and one from **either** Section B **or** Section C.

Mark allocation for **each** Project [10 marks]:

|                                |                 |
|--------------------------------|-----------------|
| Overall format                 | 1 mark          |
| Content                        | 4 marks         |
| Findings                       | 2 marks         |
| Viva-voce based on the Project | 3 marks         |
| <b>Total</b>                   | <b>10 marks</b> |

### List of suggested assignments for Project Work:

#### Section A

1. Using a Venn diagram, find the number of subsets of a given set and verify that if a set has 'n' number of elements, the total number of subsets is  $2^n$ .
  2. Verify that for two sets A and B,  $n(A \times B) = pq$ , where  $n(A) = p$  and  $n(B) = q$ , the total number of relations from A to B is  $2^{pq}$ .
  3. Using Venn diagram, verify the distributive law for three given non-empty sets A, B and C.
  4. Identify distinction between a relation and a function with suitable examples and illustrate graphically.
  5. Illustrate with the help of a model, the values of sine and cosine functions for different angles which are multiples of  $\pi/2$  and  $\pi$ .
  6. Using argand plane, interpret geometrically, the meaning of  $i = \sqrt{-1}$  and its integral powers.
  7. Draw the graph of quadratic function  $f(x) = ax^2 + bx + c$ . From the graph find maximum/minimum value of the function. Also determine the sign of the expression.
8. Construct a Pascal's triangle to write a binomial expansion for a given positive integral exponent.
  9. Obtain a formula for the sum of the squares/sum of cubes of 'n' natural numbers.
  10. Obtain the equation of the straight line in the normal form, for  $\alpha$  (the angle between the perpendicular to the line from the origin and the x-axis) for each of the following, on the same graph:
    - (i)  $\alpha < 90^\circ$
    - (ii)  $90^\circ < \alpha < 180^\circ$
    - (iii)  $180^\circ < \alpha < 270^\circ$
    - (iv)  $270^\circ < \alpha < 360^\circ$
  11. Identify the variability and consistency of two sets of statistical data using the concept of coefficient of variation.
  12. Construct the tree structure of the outcomes of a random experiment, when elementary events are not equally likely. Also construct a sample space by taking a suitable example.

#### Section B

13. Construct different types of conics by PowerPoint Presentation, or by making a model, using the concept of double cone and a plane.
14. Use focal property of ellipse to construct ellipse.
15. Use focal property of hyperbola to construct hyperbola.
16. Write geometrical significance of X coordinate, Y coordinate, and Z coordinate in space. Using the above, find the distance of the point in space from x-axis/y-axis/z-axis. Explain the above using a three-dimensional model/ power point presentation.
17. Obtain truth values of compound statements of the type  $p \wedge q$  by using switch connection in series.
18. Obtain truth values of compound statements of the type  $p \vee q$  by using switch connection in parallel.

### **Section C**

19. Find median from the point of intersection of cumulative frequency curves (less than and more than cumulative frequency curves).
20. Identify the purchasing power using the concept of cost of living index number.
21. Identify the purchasing power using the concept of weighted aggregate price index number.
22. Calculate moving averages with the given even Periodicity. Plot them and as well as the original data on the same graph.