# CISCE VIRTUAL LEARNING SERIES

## LESSON: MATHEMATICS

### TRIGONOMETRIC IDENTITIES (SESSION 1)

**October 14th, 2020**

Response to Questions posed by students during the live Lesson:

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<td>1.</td>
<td>Can we prove the standard identity ( \sec^2 \theta - \tan^2 \theta = 1 ) [1] by using ( \sin^2 \theta + \cos^2 \theta = 1 )?</td>
<td>YES</td>
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\[
\text{LHS} = \sec^2 \theta - \tan^2 \theta \\
= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\
= \frac{1 - \sin^2 \theta}{\cos^2 \theta} \\
= \frac{\cos^2 \theta}{\cos^2 \theta} \\
= 1
\]

2.  **Prove that**: \( \sin^4 \theta + \cos^4 \theta = 1 - 2\sin^2 \theta \cos^2 \theta \)

Can we solve the sum by starting directly from a standard identity instead of starting from LHS or RHS?

Yes

\[\text{We know } \sin^2 \theta + \cos^2 \theta = 1\]

Squaring both sides

\[(\sin^2 \theta + \cos^2 \theta)^2 = (1)^2\]

\[\Rightarrow \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta = 1\]

\[\Rightarrow \sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta\]

3.  Given to prove \( \cos^2 \theta (1 + \tan^2 \theta) = 1 \)

If we prove it by taking any standard angle will that be correct?

No. It will be a verification only, not proof.

\[
\text{LHS} = \cos^2 \theta (1 + \tan^2 \theta) \\
= \cos^2 \theta (\sec^2 \theta) \\
= \cos^2 \theta \left( \frac{1}{\cos^2 \theta} \right) = 1 \text{ RHS}
\]
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| 4.    | How do we prove \((1 - \tan A)^2 + (1 + \tan A)^2 = 2\sec^2 A\)? | LHS = \((1 - \tan A)^2 + (1 + \tan A)^2\)  
\[= (1 - 2\tan A + \tan^2 A) + (1 + 2\tan A + \tan^2 A)\]  
\[= 1 - 2\tan A + \tan^2 A + 1 + 2\tan A + \tan^2 A\]  
\[= 2 + 2\tan^2 A\]  
\[= 2(1 + \tan^2 A)\]  
\[= 2\sec^2 A\]  
\[= \text{RHS.}\] |
| 5.    | How do we prove \(\tan^2 x (1 + \cot^2 x) = \frac{1}{1-\sin^2 x}\)? | LHS = \(\tan^2 x (1 + \cot^2 x)\)  
\[= \tan^2 x + \tan^2 x \cot^2 x\]  
\[= \tan^2 x + \tan^2 x \cdot \frac{1}{\tan^2 x}\]  
\[= \tan^2 x + 1\]  
\[= \sec^2 x\]  
\[= \frac{1}{\cos^2 x}\]  
\[= \frac{1}{1-\sin^2 x} = \text{RHS}\] |
<p>| 6.    | Are we allowed to prove all identities by using a right angled triangle and applying Pythagoras Theorem? | It is not advisable to do so, except for standard identities. |
| 7.    | Do we get sums to prove the standard identities? | Yes, you may be asked. |
| 8.    | Is it necessary to prove from LHS to RHS or the reverse may also be done? | You may work out from any side. More detail will be given in the second session. |
| 9.    | Find the minimum value of (5\cos A + 12\sin A + 12). | These sums are not a part of the scope of your syllabus. You will learn to solve these sums in higher classes. |</p>
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<td>10.</td>
<td>In a problem of trigonometric identity can the value of angle $\theta$ be greater than $90^\circ$?</td>
<td>As per your syllabus the angle, say $\theta$, to be considered is such that, $0^\circ \leq \theta \leq 90^\circ$. But the trigonometric identities are true, irrespective of the value of the angle if the functions involved are defined.</td>
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