

## MATHEMATICS (860)

### Aims:

1. To enable candidates to acquire knowledge and to develop an understanding of the terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics at the Senior Secondary stage.
2. To develop the ability to apply the knowledge and understanding of Mathematics to unfamiliar situations or to new problems.
3. To develop an interest in Mathematics.
4. To enhance ability of analytical and rational thinking in young minds.
5. To develop skills of -
  - (a) Computation.
  - (b) Logical thinking.
  - (c) Handling abstractions.
  - (d) Generalizing patterns.
  - (e) Solving problems using multiple methods.
  - (f) Reading tables, charts, graphs, etc.
6. To develop an appreciation of the role of Mathematics in day-to-day life.
7. To develop a scientific attitude through the study of Mathematics.

A knowledge of Arithmetic, Basic Algebra (Formulae, Factorization etc.), Basic Trigonometry and Pure Geometry is assumed.

As regards to the standard of algebraic manipulation, students should be taught:

- (i) To check every step before proceeding to the next particularly where minus signs are involved.
- (ii) To attack simplification piecemeal rather than en block.
- (iii) To observe and act on any special features of algebraic form that may be obviously present.

## CLASS XI

The syllabus is divided into **three** sections A, B and C.

Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **EITHER** Section B **OR** Section C.

There will be one paper of **three** hours duration of 100 marks.

**Section A (80 Marks):** Candidates will be required to attempt **all** questions. Internal choice will be provided in three questions of four marks each and two questions of six marks each.

**Section B/ Section C (20 Marks):** Candidates will be required to attempt **all** questions **EITHER** from Section B or Section C. Internal choice will be provided in two questions of four marks each.

S.No.	UNIT	TOTAL WEIGHTAGE
<b>SECTION A: 80 Marks</b>		
1.	Sets and Functions	22 Marks
2.	Algebra	34 Marks
3.	Coordinate Geometry	8 Marks
4.	Calculus	8 Marks
5.	Statistics & Probability	8 Marks
<b>SECTION B: 20 marks</b>		
6.	Conic Section	12 Marks
7.	Introduction to Three Dimensional Geometry	4 Marks
8.	Mathematical Reasoning	4 Marks
<b>OR</b>		
<b>SECTION C: 20 Marks</b>		
9.	Statistics	6 Marks
10.	Correlation Analysis	6 Marks
11.	Index Numbers & Moving Averages	8 Marks
	<b>Total</b>	<b>100 Marks</b>

## SECTION A

### 1. Sets and Functions

#### (i) Sets

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Practical problems on union and intersection of two and three sets. Difference of sets. Complement of a set. Properties of Complement of Sets.

#### (ii) Relations & Functions

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto  $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ ). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Function as a type of mapping, types of functions (one to one, many to one, onto, into) domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotient of functions.

- *Sets: Self-explanatory.*
  - **Basic concepts of Relations and Functions**
    - *Ordered pairs, sets of ordered pairs.*
    - *Cartesian Product (Cross) of two sets, cardinal number of a cross product.*
- Relations as:*
- *an association between two sets.*
  - *a subset of a Cross Product.*
  - *Domain, Range and Co-domain of a Relation.*
- Functions:*
- *As special relations, concept of writing "y is a function of x" as  $y = f(x)$ .*

- *Introduction of Types: one to one, many to one, into, onto.*
- *Domain and range of a function.*
- *Sketches of graphs of exponential function, logarithmic function, modulus function, step function and rational function.*

#### (iii) Trigonometry

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity  $\sin^2 x + \cos^2 x = 1$ , for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing  $\sin(x \pm y)$  and  $\cos(x \pm y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$  &  $\cos y$  and their simple applications. Deducing the identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

Identities related to  $\sin 2x$ ,  $\cos 2x$ ,  $\tan 2x$ ,  $\sin 3x$ ,  $\cos 3x$  and  $\tan 3x$ . General solution of trigonometric equations of the type  $\sin y = \sin a$ ,  $\cos y = \cos a$  and  $\tan y = \tan a$ . Properties of triangles (proof and simple applications of sine rule cosine rule and area of triangle).

#### • **Angles and Arc lengths**

- *Angles: Convention of sign of angles.*
- *Magnitude of an angle: Measures of Angles; Circular measure.*
- *The relation  $S = r\theta$  where  $\theta$  is in radians. Relation between radians and degree.*

- Definition of trigonometric functions with the help of unit circle.
- Truth of the identity  $\sin^2 x + \cos^2 x = 1$
- **NOTE:** Questions on the area of a sector of a circle are required to be covered.

• **Trigonometric Functions**

- Relationship between trigonometric functions.
- Proving simple identities.
- Signs of trigonometric functions.
- Domain and range of the trigonometric functions.
- Trigonometric functions of all angles.
- Periods of trigonometric functions.
- Graphs of simple trigonometric functions (only sketches).

**NOTE:** Graphs of  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  and  $\cot x$  are to be included.

• **Compound and multiple angles**

- Addition and subtraction formula:  $\sin(A \pm B)$ ;  $\cos(A \pm B)$ ;  $\tan(A \pm B)$ ;  $\tan(A + B + C)$  etc., Double angle, triple angle, half angle and one third angle formula as special cases.
- Sum and differences as products  

$$\sin C + \sin D = 2 \sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right), \text{ etc.}$$
- Product to sum or difference i.e.  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$  etc.

**Trigonometric Equations**

- Solution of trigonometric equations (General solution and solution in the specified range).
- Equations expressible in terms of  $\sin \theta = 0$  etc.
- Equations expressible in terms i.e.

$\sin \theta = \sin \alpha$  etc.

- Equations expressible multiple and sub-multiple angles i.e.  $\sin^2 \theta = \sin^2 \alpha$  etc.
- Linear equations of the form  $a \cos \theta + b \sin \theta = c$ , where  $|c| \leq \sqrt{a^2 + b^2}$  and  $a, b \neq 0$
- Properties of  $\Delta$

Sine formula:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ ;

Cosine formula:

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ , etc

Area of triangle:  $\Delta = \frac{1}{2} bc \sin A$ , etc

Simple applications of the above.

**2. Algebra**

(i) Principle of Mathematical Induction

Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

Using induction to prove various summations, divisibility and inequalities of algebraic expressions only.

(ii) Complex Numbers

Introduction of complex numbers and their representation, Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Square root of a complex number. Cube root of unity.

- Conjugate, modulus and argument of complex numbers and their properties.
- Sum, difference, product and quotient of two complex numbers additive and multiplicative inverse of a complex number.

- Locus questions on complex numbers.
- Triangle inequality.
- Square root of a complex number.
- Cube roots of unity and their properties.

(iii) Quadratic Equations

Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients).

- Use of the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In solving quadratic equations.

- Equations reducible to quadratic form.
- Nature of roots
  - Product and sum of roots.
  - Roots are rational, irrational, equal, reciprocal, one square of the other.
  - Complex roots.
  - Framing quadratic equations with given roots.

**NOTE:** Questions on equations having common roots are to be covered.

- **Quadratic Functions.**

Given  $\alpha, \beta$  as roots then find the equation whose roots are of the form  $\alpha^3, \beta^3$ , etc.

Case I:  $a > 0$ 

- Real roots
- Complex roots
- Equal roots

Case II:  $a < 0$ 

- Real roots
- Complex roots,
- Equal roots

Where 'a' is the coefficient of  $x^2$  in the equations of the form  $ax^2 + bx + c = 0$ .

Understanding the fact that a quadratic expression (when plotted on a graph) is a parabola.

- **Sign of quadratic**

Sign when the roots are real and when they are complex.

- **Inequalities**

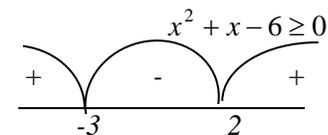
- Linear Inequalities

Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical representation of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.

Self-explanatory.

- Quadratic Inequalities

Using method of intervals for solving problems of the type:



A perfect square e.g.  $x^2 - 6x + 9 \geq 0$ .

- Inequalities involving rational expression of type

$$\frac{f(x)}{g(x)} \leq a \text{ et.c to be covered}$$

(iv) Permutations and Combinations

Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of formulae for  ${}^n P_r$  and  ${}^n C_r$  and their connections, simple application.

- Factorial notation  $n!$ ,  $n! = n(n-1)!$
- Fundamental principle of counting.
- Permutations
  - ${}^n P_r$  ..
  - Restricted permutation.
  - Certain things always occur together.

- Certain things never occur.
- Formation of numbers with digits.
- Word building - repeated letters - No letters repeated.
- Permutation of alike things.
- Permutation of Repeated things.
- Circular permutation – clockwise counterclockwise – Distinguishable / not distinguishable.

• Combinations

- ${}^n C_r, {}^n C_n = 1, {}^n C_0 = 1, {}^n C_r = {}^n C_{n-r}, {}^n C_x = {}^n C_y$ , then  $x + y = n$  or  $x = y, {}^{n+1} C_r = {}^n C_{r-1} + {}^n C_r$ .
- When all things are different.
- When all things are not different.
- Mixed problems on permutation and combinations.

(v) Binomial Theorem

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

- Significance of Pascal's triangle.
- Binomial theorem (proof using induction) for positive integral powers,

$$\text{i.e. } (x + y)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} y + \dots + {}^n C_n y^n.$$

Questions based on the above.

(vi) Sequence and Series

Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of first  $n$  terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Formulae for the following special sums  $\sum n, \sum n^2, \sum n^3$ .

- Arithmetic Progression (A.P.)

- $T_n = a + (n - 1)d$

- $S_n = \frac{n}{2} \{2a + (n - 1)d\}$
- Arithmetic mean:  $2b = a + c$
- Inserting two or more arithmetic means between any two numbers.
- Three terms in A.P. :  $a - d, a, a + d$
- Four terms in A.P.:  $a - 3d, a - d, a + d, a + 3d$

• Geometric Progression (G.P.)

- $T_n = ar^{n-1}, S_n = \frac{a(r^n - 1)}{r - 1}$ ,
- $S_\infty = \frac{a}{1 - r}; |r| < 1$  Geometric Mean,  $b = \sqrt{ac}$
- Inserting two or more Geometric Means between any two numbers.
- Three terms are in G.P.  $ar, a, ar^{-1}$
- Four terms are in GP  $ar^3, ar, ar^{-1}, ar^{-3}$

• Arithmetico Geometric Series

Identifying series as A.G.P. (when we substitute  $d = 0$  in the series, we get a G.P. and when we substitute  $r = 1$  the A.P).

- Special sums  $\sum n, \sum n^2, \sum n^3$

Using these summations to sum up other related expression.

3. Coordinate Geometry

(i) Straight Lines

Brief recall of two dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope- intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

- Basic concepts of Points and their coordinates.

- *The straight line*
  - *Slope or gradient of a line.*
  - *Angle between two lines.*
  - *Condition of perpendicularity and parallelism.*
  - *Various forms of equation of lines.*
  - *Slope intercept form.*
  - *Two point slope form.*
  - *Intercept form.*
  - *Perpendicular /normal form.*
  - *General equation of a line.*
  - *Distance of a point from a line.*
  - *Distance between parallel lines.*
  - *Equation of lines bisecting the angle between two lines.*
  - *Equation of family of lines*
  - *Definition of a locus.*
  - *Equation of a locus.*

(ii) Circles

- *Equations of a circle in:*
  - *Standard form.*
  - *Diameter form.*
  - *General form.*
  - *Parametric form.*
- *Given the equation of a circle, to find the centre and the radius.*
- *Finding the equation of a circle.*
  - *Given three non collinear points.*
  - *Given other sufficient data for example centre is  $(h, k)$  and it lies on a line and two points on the circle are given, etc.*
- *Tangents:*
  - *Condition for tangency*
  - *Equation of a tangent to a circle*

**4. Calculus**

(i) Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically.

Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to slope of tangent of the curve, Derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

• *Limits*

- *Notion and meaning of limits.*
- *Fundamental theorems on limits (statement only).*
- *Limits of algebraic and trigonometric functions.*
- *Limits involving exponential and logarithmic functions.*

**NOTE:** *Indeterminate forms are to be introduced while calculating limits.*

• *Differentiation*

- *Meaning and geometrical interpretation of derivative.*
- *Derivatives of simple algebraic and trigonometric functions and their formulae.*
- *Differentiation using first principles.*
- *Derivatives of sum/difference.*
- *Derivatives of product of functions.*
- *Derivatives of quotients of functions.*

**5. Statistics and Probability**

(i) Statistics

Measures of dispersion: range, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

- *Mean deviation about mean and median.*
- *Standard deviation - by direct method, short cut method and step deviation method.*

**NOTE:** *Mean, Median and Mode of grouped and ungrouped data are required to be covered.*

(ii) Probability

Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories studied in earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

- Random experiments and their outcomes.
- Events: sure events, impossible events, mutually exclusive and exhaustive events.
  - Definition of probability of an event
  - Laws of probability addition theorem.

SECTION B

6. Conic Section

Sections of a cone, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola.

- Conics as a section of a cone.
  - Definition of Foci, Directrix, Latus Rectum.
  - $PS = ePL$  where  $P$  is a point on the conics,  $S$  is the focus,  $PL$  is the perpendicular distance of the point from the directrix.

(i) Parabola

$e = 1, y^2 = \pm 4ax, x^2 = 4ay, y^2 = -4ax,$

$x^2 = -4ay, (y - \beta)^2 = \pm 4a(x - \alpha),$

$(x - \alpha)^2 = \pm 4a(y - \beta).$

- Rough sketch of the above.
- The latus rectum; quadrants they lie in; coordinates of focus and vertex; and equations of directrix and the axis.
- Finding equation of Parabola when Foci and directrix are given, etc.
- Application questions based on the above.

(ii) Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, e < 1, b^2 = a^2(1 - e^2)$

$\frac{(x - \alpha)^2}{a^2} + \frac{(y - \beta)^2}{b^2} = 1$

- Cases when  $a > b$  and  $a < b$ .
- Rough sketch of the above.
- Major axis, minor axis; latus rectum; coordinates of vertices, focus and centre; and equations of directrices and the axes.
- Finding equation of ellipse when focus and directrix are given.
- Simple and direct questions based on the above.
- Focal property i.e.  $SP + SP' = 2a$ .

(iii) Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, e > 1, b^2 = a^2(e^2 - 1)$

$\frac{(x - \alpha)^2}{a^2} - \frac{(y - \beta)^2}{b^2} = 1$

- Cases when coefficient  $y^2$  is negative and coefficient of  $x^2$  is negative.
- Rough sketch of the above.
- Focal property i.e.  $SP - S'P = 2a$ .
- Transverse and Conjugate axes; Latus rectum; coordinates of vertices, foci and centre; and equations of the directrices and the axes.
- General second degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ 
  - Case 1: pair of straight line if  $abc + 2fgh - af^2 - bg^2 - ch^2 = 0,$
  - Case 2:  $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0,$  then represents a parabola if  $h^2 = ab,$  ellipse if  $h^2 < ab,$  and hyperbola if  $h^2 > ab.$
- Condition that  $y = mx + c$  is a tangent to the conics, general equation of tangents, point of contact and locus problems.

## 7. Introduction to three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

- As an extension of 2-D
- Distance formula.
- Section and midpoint form

## 8. Mathematical Reasoning

Mathematically acceptable statements. Connecting words/ phrases - consolidating the understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "and", "or", "there exists" and their use through variety of examples related to the Mathematics and real life. Validating the statements involving the connecting words, Difference between contradiction, converse and contrapositive.

Self-explanatory.

## SECTION C

### 9. Statistics

- Combined mean and standard deviation.
- The Median, Quartiles, Deciles, Percentiles and Mode of grouped and ungrouped data.

### 10. Correlation Analysis

- Definition and meaning of covariance.
- Coefficient of Correlation by Karl Pearson.

If  $x - \bar{x}$ ,  $y - \bar{y}$  are small non - fractional numbers, we use

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2} \sqrt{\sum(y - \bar{y})^2}}$$

If  $x$  and  $y$  are small numbers, we use

$$r = \frac{\sum xy - \frac{1}{N} \sum x \sum y}{\sqrt{\sum x^2 - \frac{1}{N} (\sum x)^2} \sqrt{\sum y^2 - \frac{1}{N} (\sum y)^2}}$$

Otherwise, we use assumed means

$A$  and  $B$ , where  $u = x - A$ ,  $v = y - B$

$$r = \frac{\sum uv - \frac{1}{N} (\sum u)(\sum v)}{\sqrt{\sum u^2 - \frac{1}{N} (\sum u)^2} \sqrt{\sum v^2 - \frac{1}{N} (\sum v)^2}}$$

- Rank correlation by Spearman's (Correction included).

### 11. Index Numbers and Moving Averages

- Index Numbers
  - Price index or price relative.
  - Simple aggregate method.
  - Weighted aggregate method.
  - Simple average of price relatives.
  - Weighted average of price relatives (cost of living index, consumer price index).
- Moving Averages
  - Meaning and purpose of the moving averages.
  - Calculation of moving averages with the given periodicity and plotting them on a graph.
  - If the period is even, then the centered moving average is to be found out and plotted.

## CLASS XII

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S.No.	UNIT	TOTAL WEIGHTAGE
<b>SECTION A: 80 MARKS</b>		
1.	Relations and Functions	12 Marks
2.	Algebra	14 Marks
3.	Calculus	40 Marks
4.	Probability	14 Marks
<b>SECTION B: 20 MARKS</b>		
5.	Vectors	6 Marks
6.	Three - Dimensional Geometry	8/10 Marks
7.	Applications of Integrals	6/4 Marks
<b>OR</b>		
<b>SECTION C: 20 MARKS</b>		
8.	Application of Calculus	8 Marks
9.	Linear Regression	6 Marks
10.	Linear Programming	6 Marks
<b>Total</b>		<b>100 Marks</b>

## SECTION A

### 1. Relations and Functions

(i) Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

- *Relations as:*
  - *Relation on a set A*
  - *Identity relation, empty relation, universal relation.*
  - *Types of Relations: reflexive, symmetric, transitive and equivalence relation.*
- *Binary Operation: all axioms and properties*
- *Functions:*
  - *As special relations, concept of writing “y is a function of x” as  $y = f(x)$ .*
  - *Types: one to one, many to one, into, onto.*
  - *Real Valued function.*
  - *Domain and range of a function.*
  - *Conditions of invertibility.*
  - *Composite functions and invertible functions (algebraic functions only).*

(ii) Inverse Trigonometric Functions

Definition, domain, range, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

- *Principal values.*
- *$\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$  etc. and their graphs.*
- $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ .
- $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}; \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$  and similar relations for  $\cot^{-1}x, \tan^{-1}x$ , etc.

$$\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-y^2}\sqrt{1-x^2}\right)$$

$$\text{similarly } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, xy > -1$$

- *Formulae for  $2\sin^{-1}x, 2\cos^{-1}x, 2\tan^{-1}x, 3\tan^{-1}x$  etc. and application of these formulae.*

### 2. Algebra

Matrices and Determinants

(i) Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order upto 3). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists (here all matrices will have real entries).

(ii) Determinants

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

- Types of matrices ( $m \times n$ ;  $m, n \leq 3$ ), order; Identity matrix, Diagonal matrix.
- Symmetric, Skew symmetric.
- Operation – addition, subtraction, multiplication of a matrix with scalar, multiplication of two matrices (the compatibility).

$$\text{E.g. } \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = AB(\text{say}) \text{ but } BA \text{ is}$$

not possible.

- Singular and non-singular matrices.
- Existence of two non-zero matrices whose product is a zero matrix.

- Inverse ( $2 \times 2, 3 \times 3$ )  $A^{-1} = \frac{\text{Adj}A}{|A|}$

- **Martin's Rule (i.e. using matrices)**

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

Problems based on above.

**NOTE 1:** The conditions for consistency of equations in two and three variables, using matrices, are to be covered.

**NOTE 2:** Inverse of a matrix by elementary operations to be covered.

- **Determinants**

- Order.
- Minors.
- Cofactors.
- Expansion.
- Applications of determinants in finding the area of triangle and collinearity.
- Properties of determinants. Problems based on properties of determinants.

### 3. Calculus

- (i) Continuity, Differentiability and Differentiation. Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

- **Continuity**

- Continuity of a function at a point  $x = a$ .
- Continuity of a function in an interval.
- Algebra of continuous function.
- Removable discontinuity.

- **Differentiation**

- Concept of continuity and differentiability of  $|x|$ ,  $[x]$ , etc.
- Derivatives of trigonometric functions.
- Derivatives of exponential functions.
- Derivatives of logarithmic functions.
- Derivatives of inverse trigonometric functions - differentiation by means of substitution.
- Derivatives of implicit functions and chain rule
- e for composite functions.
- Derivatives of Parametric functions.
- Differentiation of a function with respect to another function e.g. differentiation of  $\sin x^3$  with respect to  $x^3$ .
- Logarithmic Differentiation - Finding  $dy/dx$  when  $y = x^{x^x}$ .
- Successive differentiation up to 2<sup>nd</sup> order.

**NOTE 1:** Derivatives of composite functions using chain rule.

**NOTE 2:** Derivatives of determinants to be covered.

- *L' Hospital's theorem.*
  - $\frac{0}{0}$  form,  $\frac{\infty}{\infty}$  form,  $0^0$  form,  $\infty^\infty$  form etc.
- *Rolle's Mean Value Theorem - its geometrical interpretation.*
- *Lagrange's Mean Value Theorem - its geometrical interpretation*

(ii) Applications of Derivatives

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-lifesituations).

- *Equation of Tangent and Normal*
- *Approximation.*
- *Rate measure.*
- *Increasing and decreasing functions.*
- *Maxima and minima.*
  - *Stationary/turning points.*
  - *Absolute maxima/minima*
  - *local maxima/minima*
  - *First derivatives test and second derivatives test*
  - *Point of inflexion.*
  - *Application problems based on maxima and minima.*

(iii) Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

• *Indefinite integral*

- *Integration as the inverse of differentiation.*
- *Anti-derivatives of polynomials and functions  $(ax + b)^n$ ,  $\sin x$ ,  $\cos x$ ,  $\sec^2 x$ ,  $\operatorname{cosec}^2 x$  etc .*
- *Integrals of the type  $\sin^2 x$ ,  $\sin^3 x$ ,  $\sin^4 x$ ,  $\cos^2 x$ ,  $\cos^3 x$ ,  $\cos^4 x$ .*
- *Integration of  $1/x$ ,  $e^x$ .*
- *Integration by substitution.*
- *Integrals of the type  $f'(x)[f(x)]^n$ ,  $\frac{f'(x)}{f(x)}$ .*
- *Integration of  $\tan x$ ,  $\cot x$ ,  $\sec x$ ,  $\operatorname{cosec} x$ .*
- *Integration by parts.*
- *Integration using partial fractions.*  
*Expressions of the form  $\frac{f(x)}{g(x)}$  when degree of  $f(x) <$  degree of  $g(x)$*

E.g. 
$$\frac{x+2}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$\frac{x+2}{(x-2)(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$$\frac{x+1}{(x^2+3)(x-1)} = \frac{Ax+B}{x^2+3} + \frac{C}{x-1}$$

When degree of  $f(x) \geq$  degree of  $g(x)$ ,

e.g.

$$\frac{x^2+1}{x^2+3x+2} = 1 - \left( \frac{3x+1}{x^2+3x+2} \right)$$

- Integrals of the type:

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$$

$$\text{and } \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx,$$

$$\int \sqrt{ax^2+bx+c} dx, \int (px+q)\sqrt{ax^2+bx+c} dx,$$

integrations reducible to the above forms.

$$\int \frac{dx}{a \cos x + b \sin x},$$

$$\int \frac{dx}{a+b \cos x}, \int \frac{dx}{a+b \sin x}, \int \frac{dx}{a \cos x + b \sin x + c},$$

$$\int \frac{(a \cos x + b \sin x) dx}{c \cos x + d \sin x},$$

$$\int \frac{dx}{a \cos^2 x + b \sin^2 x + c}$$

$$\int \frac{1 \pm x^2}{1+x^4} dx,$$

$$\int \frac{dx}{1+x^4}, \int \sqrt{\tan x} dx, \int \sqrt{\cot x} dx \text{ etc.}$$

- Definite Integral

- Definite integral as a limit of the sum.
- Fundamental theorem of calculus (without proof)
- Properties of definite integrals.
- Problems based on the following properties of definite integrals are to be covered.

$$\int_a^b f(x) dx = \int_a^b f(t) dt$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

where  $a < c < b$

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$

$$\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f \text{ is an even function} \\ 0, & \text{if } f \text{ is an odd function} \end{cases}$$

#### (iv) Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:  $\frac{dy}{dx} + py = q$ ,

where  $p$  and  $q$  are functions of  $x$  or constants.  $\frac{dx}{dy} + px = q$ , where  $p$  and  $q$  are

functions of  $y$  or constants.

- Differential equations, order and degree.
- Formation of differential equation by eliminating arbitrary constant(s).
- Solution of differential equations.
- Variable separable.
- Homogeneous equations.
- Linear form  $\frac{dy}{dx} + Py = Q$  where  $P$  and  $Q$  are functions of  $x$  only. Similarly for  $\frac{dx}{dy}$ .

- Solve problems of application on growth and decay.
- Solve problems on velocity, acceleration, distance and time.
- Solve population based problems on application of differential equations.
- Solve problems of application on coordinate geometry.

**NOTE 1:** Equations reducible to variable separable type are included.

**NOTE 2:** The second order differential equations are excluded.

#### 4. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

- Independent and dependent events conditional events.
- Laws of Probability, addition theorem, multiplication theorem, conditional probability.
- Theorem of Total Probability.
- Baye's theorem.
- Theoretical probability distribution, probability distribution function; mean and variance of random variable, Repeated independent (Bernoulli trials), binomial distribution – its mean and variance.

### SECTION B

#### 5. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

- As directed line segments.
- Magnitude and direction of a vector.
- Types: equal vectors, unit vectors, zero vector.
- Position vector.
- Components of a vector.
- Vectors in two and three dimensions.
- $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  as unit vectors along the  $x$ ,  $y$  and the  $z$  axes; expressing a vector in terms of the unit vectors.
- Operations: Sum and Difference of vectors; scalar multiplication of a vector.
- Section formula.
- Triangle inequalities.
- Scalar (dot) product of vectors and its geometrical significance.
- Cross product - its properties - area of a triangle, area of parallelogram, collinear vectors.
- Scalar triple product - volume of a parallelepiped, co-planarity.

**NOTE:** Proofs of geometrical theorems by using Vector algebra are excluded.

#### 6. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

- Equation of  $x$ -axis,  $y$ -axis,  $z$  axis and lines parallel to them.
- Equation of  $xy$  - plane,  $yz$  - plane,  $zx$  - plane.
- Direction cosines, direction ratios.
- Angle between two lines in terms of direction cosines /direction ratios.
- Condition for lines to be perpendicular/parallel.
- Lines
  - Cartesian and vector equations of a line through one and two points.
  - Coplanar and skew lines.

- Conditions for intersection of two lines.
- Distance of a point from a line.
- Shortest distance between two lines.

**NOTE: Symmetric and non-symmetric forms of lines are required to be covered.**

- Planes
  - Cartesian and vector equation of a plane.
  - Direction ratios of the normal to the plane.
  - One point form.
  - Normal form.
  - Intercept form.
  - Distance of a point from a plane.
  - Intersection of the line and plane.
  - Angle between two planes, a line and a plane.
  - Equation of a plane through the intersection of two planes i.e.  $P_1 + kP_2 = 0$ .

## 7. Application of Integrals

Application in finding the area bounded by simple curves and coordinate axes. Area enclosed between two curves.

- Application of definite integrals - area bounded by curves, lines and coordinate axes is required to be covered.
- Simple curves: lines, circles/ parabolas/ ellipses, polynomial functions, modulus function, trigonometric function, exponential functions, logarithmic functions

## SECTION C

## 8. Application of Calculus

Application of Calculus in Commerce and Economics in the following:

- Cost function,
- average cost,
- marginal cost and its interpretation
- demand function,
- revenue function,
- marginal revenue function and its interpretation,
- Profit function and breakeven point.

- Rough sketching of the following curves: AR, MR, R, C, AC, MC and their mathematical interpretation using the concept of maxima & minima and increasing- decreasing functions.

*Self-explanatory*

**NOTE: Application involving differentiation, integration, increasing and decreasing function and maxima and minima to be covered.**

## 9. Linear Regression

- Lines of regression of x on y and y on x.
- Scatter diagrams
- The method of least squares.
- Lines of best fit.
- Regression coefficient of x on y and y on x.
- $b_{xy} \times b_{yx} = r^2$ ,  $0 \leq b_{xy} \times b_{yx} \leq 1$
- Identification of regression equations
- Angle between regression line and properties of regression lines.
- Estimation of the value of one variable using the value of other variable from appropriate line of regression.

*Self-explanatory*

## 10. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions(bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

*Introduction, definition of related terminology such as constraints, objective function, optimization, advantages of linear programming; limitations of linear programming; application areas of linear programming; different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimum feasible solution.*